Cooperative Control Technique for Quadrotor UAV Trajectory Tracking

Wesam M. Jasim[1]
[1]College of Computer Science and Information Technology, University of Anbar, Iraq

Abstract — **Recently, the quadrotor Unmanned Aerial Vehicle UAV has been developed to be useful in variety of applications in both military and civil. Control of this type of vehicles still the main problem faced the researchers. In this paper, two cooperative controllers were implemented to control the UAV pose (3D trajectory tracking) in two loops. The first controller is the Proportional Derivative PD controller which used to stabilize the UAV (attitude control or control the orientation angles) in the inner loop. The second controller is the Linear Quadratic Regulator LQR controller which used to control the position of the UAV in the outer loop. Controlling the x and y axes in the first loop were used in the second loop to control their angles. These two controllers were designed based on the UAV linearized model about the hovering condition taking into account the dynamic model constraints and external disturbances. Simulation results shows that the proposed control strategy control the UAV effectively and overcome the dynamic model constraints and external disturbances in terms of small mean square error.**

Keywords: quadrotor UAV, PD control, LQR control.

1. Introduction

In the last decades the design of the UAV was widely increased. Different UAV configurations have been built to perform different dangerous tasks. One of the most popular and easy to control is the quadrotor configuration. In addition, the application of this type of UAV was varied to perform military and civilian tasks. Nevertheless, the widely growth in technology was affect the development of the UAV by adding advances sensors, motors, electronics and Global Positioning System GPS[1].

Control of the quadrotor has been the focus of extensive research across the wide range of applications. Several control techniques were designed for single or multi quadrotors such as, PID, Backstepping BS, Genetic Algorithm with BS GA-BS, and improved GA-BS controllers are proposed and simulated in [2] to prove that the quadrotor can be controlled the angle and speed with the consideration of the deviation of the quadrotor center of gravity. The simulation results indicated that the improved GA-BS controller outperform the other three controllers. Authors in [3] applied a model independent controller to control the UAV with a semi autopilot. The real time results of the proposed controller demonstrate it is effectiveness when compared with that of PID controller. Ghulam E. M. A**.** et al. [4] proposed the fuzzy based sliding mode control technique in two loop control for both attitude and position. This technique led to a small root mean square error with higher deviation in takeoff. A hybrid PID backstepping with forwarded control strategy is discussed when applied to a quadrotor with high nonlinearity dynamic model. Simulink software is designed and implemented to measure the controller performance [5]. A discrete PID dependent control technique is proposed to control a large quadrotor (X-4 flyer, more than 3Kg) to be used in load carrying in [6]. The controller is a single input single output controller. Indoor and outdoor practical tests show that the implemented prototype is stabilized and the performance is validated in low speed. Jiahao Zhao [7] applied a cascade PID controller for the attitude stabilization and the position of the UAV quadrotor. The quadrotor dynamic system is nonlinear and the environment has some obstacles. The proposed control technique tested in line flight first then in target position with obstacle avoidance. Simulation results show that the controller derive the vehicle to the position successfully.

A reinforcement trained neural network control approach is proposed in [8] to stabilize the vehicle while it in air to move up and down to track a way point path with high initial velocity. The resultant steady state error was less than two cm. A sliding mode with backstepping control technique is suggested to control the Quadrotor UAV and track a certain path in [9]. The controller stability is analyzed based on Lyapunov theory. To demonstrate the controller robustness a white Gaussian noise is considered as external disturbance with quadrotor the aerodynamic moment. The mean results show the effectiveness of the proposed technique when it compared with that of LQR and backstepping techniques. Authors in [10] employed an integral backstepping control algorithm to solve the quadrotor control problem while tracking a predefined path. The controller stability is guaranteed based on Lyapunov function. The simulation results express the robustness of the controller. In this paper, a combination of two controllers named PD and LQR to solve the problem of quadrotor stabilization and tracking with vehicle dynamics and external disturbance consideration.

The rest of the paper is organized as follows; section 2 demonstrates the vehicle mathematical model. Section 3 illustrates the proposed control strategy design and analysis. Section 4 discusses the obtained simulation results, while section 5 concludes the findings and suggests the future work.

DOI:http://doi.org/10.24086/cocos2024/paper.1561

2. Quadrotor Modeling

Figure (1) shows the Ascending Technology Hummingbird with Autopilot that it is parameters is used in this work. The full nonlinear quadrotor model based on Newton-Euler formulation method is illustrated in eq.(1), the full derivation of the model can be seen in [11].

Figure (1). Ascending Technology

$$\begin{cases} \ddot{x} = (-\cos\varphi\cos\psi\sin\theta - \sin\varphi\sin\psi)\frac{1}{m}F \\ \ddot{y} = (-\cos\varphi\sin\psi\sin\theta + \sin\varphi\cos\psi)\frac{1}{m}F \\ \ddot{z} = g - (\cos\varphi\cos\theta)\frac{1}{m}F \\ \ddot{\varphi} = \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{I_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}\tau_\varphi \\ \ddot{\theta} = \dot{\varphi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{I_r}{I_y}\dot{\varphi}\Omega + \frac{l}{I_y}\tau_\theta \\ \ddot{\psi} = \dot{\theta}\dot{\varphi}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}\tau_\psi \end{cases}$$

(1)

Where Ω is the propeller angular velocity.

The full mathematical model of the Quadrotor UAV is represented by eq. (1) which is rewritten in a state space model of $\dot{X} = f(X, U)$ where X is the state vector as in eq.(2) and eq.(3).

$$X = [x\ \dot{x}\ y\ \dot{y}\ \ z\ \dot{z}\ \varphi\ \dot{\varphi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}]^T$$

(2)

$$\begin{cases} x_1 = x \\ x_2 = \dot{x}_1 = \dot{x} \\ x_3 = y \\ x_4 = \dot{x}_3 = \dot{y} \\ x_5 = z \\ x_6 = \dot{x}_5 = \dot{z} \\ x_7 = \varphi \\ x_8 = \dot{x}_7 = \dot{\varphi} \\ x_9 = \theta \\ x_{10} = \dot{x}_9 = \dot{\theta} \\ x_{11} = \psi \\ x_{12} = \dot{x}_{11} = \dot{\psi} \end{cases}$$

(3)

and $U_i\ (i = 1,2,3,4)$ are the inputs and they can be written as in eq.(4)

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(\Omega_4^2 - \Omega_2^2) \\ U_3 = b(\Omega_3^2 - \Omega_1^2) \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \\ \Omega = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3 \end{cases}$$

(4)

$$U = [U_1\ \ U_2\ \ U_3\ \ U_4]^T$$

(5)

From eq.(3) and eq.(1) we get eq.(6) as a state variables where the first six states are for translations and the other are for the attitude states.

$$f(X,U) = \ddot{X} = \begin{cases} x_2 \\ (-\cos x_7 \sin x_9 \cos x_{11} - \sin x_7 \sin x_{11})\frac{U_1}{m} = u_x\frac{U_1}{m} \\ x_4 \\ (-\cos x_7 \sin x_9 \sin x_{11} + \sin x_7 \cos x_{11})\frac{U_1}{m} = u_y\frac{U_1}{m} \\ x_6 \\ g - (\cos x_7 \cos x_9)\frac{U_1}{m} = u_z \\ x_8 \\ x_{12}x_{10}\left(\frac{I_y - I_z}{I_x}\right) - \frac{I_r}{I_x}x_{10}\Omega + \frac{l}{I_x}U_2 \\ x_{10} \\ x_{12}x_8\left(\frac{I_z - I_x}{I_y}\right) - \frac{I_r}{I_y}x_8\Omega + \frac{l}{I_y}U_3 \\ x_{12} \\ x_{10}x_8\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}U_4 \end{cases}$$

(6)

Eq.(6) illustrate the vehicle nonlinear model. The most important requirement for LQR control design is the system is linear. Therefore; the system of eq.(6) should be linearized around a certain point and conditions. For the vertical takeoff and landing vehicle as the quadrotor the operating point is the hovering point. While conditions are appeared based on the hovering point which are; the three attitude angles, their angular velocities, and the three linear velocities of the position parameters are equal to zero. Then the system of eq. (6) with the use of the state's definitions in eq.(3) can be linearized as in eq. (7).

DOI:http://doi.org/10.24086/cocos2024/paper.1561

$$f(X,U) = \begin{cases} \dot{x} \\ \theta g \\ \dot{y} \\ -\varphi g \\ \dot{z} \\ -g + \frac{U_1}{m} \\ \dot{\varphi} \\ \frac{1}{I_x} U_2 \\ \dot{\theta} \\ \frac{1}{I_y} U_3 \\ \dot{\psi} \\ \frac{1}{I_z} U_4 \end{cases}$$

(7)

3. Control Design

Two controllers are implemented in this work to control the quadrotor UAV; the first one is the PD controller which be used to control the attitude angles of the quadrotor (stabilization of the vehicle). While the second controller is the LQR controller which be used to control the position and the altitude of the vehicle. The two controllers' derivations are in the next subsections.

3.1 LQR Controller

LQR is a fixed parameter modern optimal control state feedback technique which designed and analyzed based on the system state space of the form eq. (8).

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \quad (8)$$

The LQR controller goal is to find the optimal controller parameters K such that the state feedback control rule eq. (9) minimizes the cost function presented in eq. (10) [12].

$$u = -KX = -R^{-1}B^T PX \quad (9)$$

$$I = \int_0^\infty (X^T QX + u^T Ru)dt \quad (10)$$

Where A, B, C are the linearized system matrices, Q is a matrix represent the state cost and it is value greater than or equal zero and R is a matrix represent the control cost of value greater than zero, and P can be obtained from the Ricatti equation (11).

$$\dot{P}(t) + P(t)A + A^T P(t) - P(t)BR^{-1}B^T P(t) + Q = 0 \quad (11)$$

Then K can be obtained based on the control law of eq. (12).

$$K = lqr(A, B, Q, R) \quad (12)$$

This controller is designed to control the altitude and the position of the vehicle as in the next subsections.

3.1.1 Altitude and position control

The control input U_1 can be obtained from the LQR controller with respect to altitude desired z_d as in state space eq. (13).

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_z \quad (13)$$

While the position control can be obtained based on the LQR controller with respect to x_d and y_d as in state space eq. (14)

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g & 0 \\ 0 & 0 \\ 0 & -g \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

The obtained θ_d and φ_d from the attitude control part is used to control the position axis.

3.2 Attitude Controller

A PD controller was implemented to control the attitude angles(φ, θ, ψ). This controller shows a good performance in terms of minimum errors and stability. The control law to find the U_2 U_3 U_4 is as in eq. (15), (16), and (17).

$$U_2 = k_p(\theta_d - \theta) + k_d(\dot{\theta}_d - \dot{\theta}) \quad (15)$$

$$U_3 = k_p(\varphi_d - \varphi) + k_d(\dot{\varphi}_d - \dot{\varphi}) \quad (15)$$

$$U_4 = k_p(\psi_d - \psi) + k_d(\dot{\psi}_d - \dot{\psi}) \quad (15)$$

Where θ_d and φ_d are calculated based on eq.(6).

4. Simulation Results

The tracking performance of the cooperative controller is tested using a three dimensions path defined in eq. (16). The vehicle specifications used in the simulation are listed in table (1). The system is simulated using MATLAB 2018b.

$$\begin{cases} x_d = 5 * t \\ y_d = 5 * cost \\ z_d = t \\ \psi_d = 0 \end{cases} \quad (16)$$

with zero initial condition.

Table (1). Quadrotor Parameters

Parameter	Definition	Value	Units
m	Vehicle Mass	0.5	Kg
g	Gravity	9.81	m/s^2
l	Arm Length	0.17	m
I_r	Rotor Inertia	0.000044	$Kg.m^2$
I_x	Roll Inertia	0.0044	$Kg.m^2$
I_y	Pitch Inertia	0.0044	$Kg.m^2$
I_z	Yaw Inertia	0.0088	$Kg.m^2$

The simulation results were achieved by applying both the LQR controller for altitude and position and PD controller for attitude to stabilize and track the predefined trajectory in eq. (16) are shown in figures (2), (3), and (4), the 3D trajectory, altitude and positions, and the attitude angles respectively. These results obtained under four cases; no disturbance, force disturbance on the altitude and position as

DOI:http://doi.org/10.24086/cocos2024/paper.1561

$d_x = 10N\ at\ t = 20s, d_y = 10N\ at\ t = 30s, and\ d_z =$, $+10\%$ model parameter uncertainty, and -10% model parameter uncertainty. The LQR controller gains were achieved to be $K_x = diag(70,50)$, $K_y = diag(12,6)$, and $K_z = diag(90,50)$ for x, y, and z directions respectively. While the PD controller gains were achieved to be
$k_{p_\theta} = 50, k_{d_\theta} = 8, k_{p_\varphi} = 50, k_{d_\varphi} = 8.09, k_{p_\psi} = 50, an$
for roll, pitch, and yaw angle respectively.

The results show that the cooperative control technique can drive the vehicle to track the trajectory with small error, less than 3 cm, while the attitude angles can track their desired values successfully, in all four cases. It obviously that the controller can recover the change in the model parameter uncertainty and overcome the effect of the external disturbances. Moreover; it can drive the vehicle to the desired trajectory smoothly and safely in all the four circumstances.

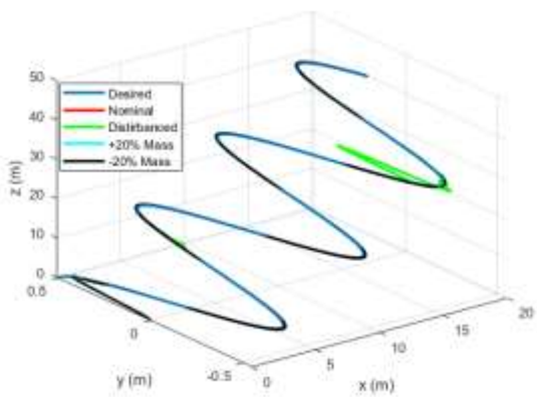

Figure (2). Global 3D Trajectory Tracking in All Circumstances

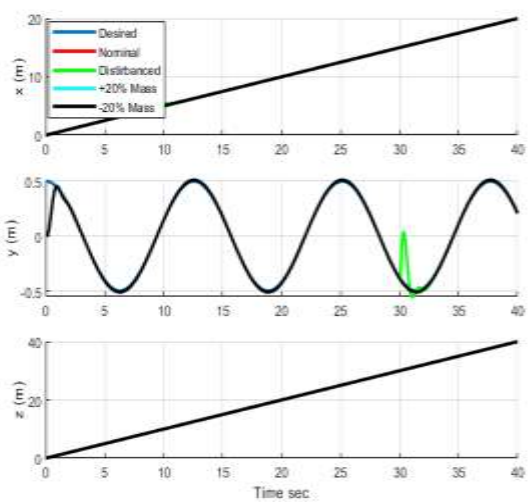

Figure (3). Position Trajectory Tracking in All Circumstances

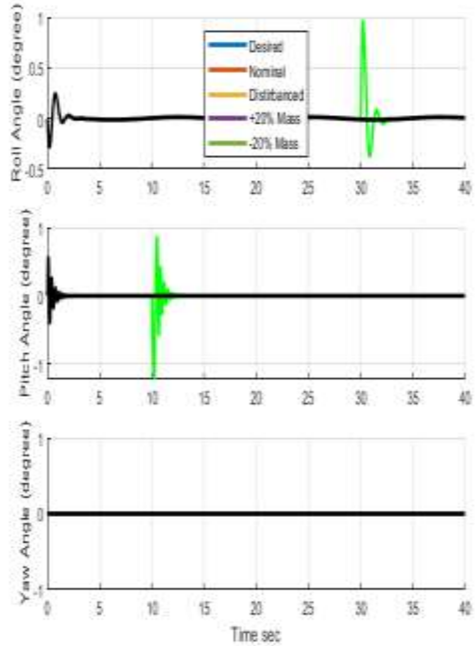

Figure (4). Angle Response in All Circumstances

5. Conclusions

This paper provides two cooperative controllers for unmanned aerial vehicle trajectory tracking. It is shown that the LQR controller is used for position control, while the PD controller is used for attitude stabilization. The simulation results were achieved with model constraints change and external disturbance effects to further examine the robustness of the controllers. Potential future work could involve the recovery the effect of stopping one motor of the vehicle during flight by developing a nonlinear control strategy that can help to examine more suitable controller in different circumstances.

References

[1] Muhammed Maaruf, Magdi Sadek Mahmoud, and Alfian Ma'arif, "A Survey of Control Methods for Quadrotor UAV," *International Journalof Robotics and Control Systems,*vol. 2, no. 4, pp. 652-665, 2022.

[2] Lirong Zhou, Anton Pljonkin, and Pradeep Kumar Singh, "Modeling and PID Control of Quadrotor UAV based on Machine Learning." *jisys*, vol. 31, pp. 1112-1122, 2022.

[3] Guoyuan Qi, Shengli Ma, Xitong Guo, Xia Li, and Jianchuan Guo, "High-Order Differential Feedback control for Quadrotor UAV: Theory and Experimentation," *Electronics*, vol. 9, pp. 1-19, 2020.

[4] G. E. Mustafa Abro, Z. A. Ali, S. A. Zulkifli, and V. S. Asirvadam, "Performance Evaluation of Different Control Methods for an Underactuated Quadrotor Unmanned Aerial Vehicle (QUAV) with Position Estimator and Disturbance Observer," *Math. Probl. Eng.*, vol. 2021, 2021, doi: 10.1155/2021/8791620.

[5] F. Ouendi, S. Houti, and M. Tadjine, "Comparison of quadrotor performance using forwarding and PID-backstepping control," *Nonlinear Dyn. Syst. Theory*, vol. 19, no. 3, pp. 427–443, 2019.

[6] P. Pounds, R. Mahony, and P. Corke, "Modelling and control of a large quadrotor robot," *Control Eng. Pract.*, vol. 18, no. 7, pp. 691–699, 2010, doi: 10.1016/j.conengprac.2010.02.008.

[7] Jiahao Zhao, "Quadrotor's Modeling and Control System Design based on PID Control," *IOP Publishing, EIMM-2022,* no.2483, pp.

1–6, 2022.

[8] J. Hwangbo, I. Sa, R. Siegwart, and M. Hutter, "Control of a Quadrotor with Reinforcement Learning," *IEEE Robot. Autom. Lett.*, vol. 2, no. 4, pp. 2096–2103, 2017, doi: 10.1109/LRA.2017.2720851.

[9] S. Nadda and A. Swarup, "Tracking control design for quadrotor unmanned aerial vehicle," *Def. Sci. J.*, vol. 67, no. 3, pp. 245–253, 2017, doi: 10.14429/dsj.67.9536.

[10] A. Bouchaib, R. Taleb, A. Massoum, and S. Mekhilef, "Geometric control of quadrotor UAVs using integral backstepping," *Indones. J. Electr. Eng. Comput. Sci.*, vol. 22, no. 1, pp. 53–61, 2021, doi: 10.11591/ijeecs.v22.i1.pp53-61.

[11] W. Jasim and D. Gu, "Robust Team Formation Control for Quadrotors," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 4, pp. 1516–1523, 2018, doi: 10.1109/TCST.2017.2705072.

[12] Y. Alothman, W. Jasim, and D. Gu, "Quad-rotor lifting-transporting cable-suspended payloads control," *2015 21st Int. Conf. Autom. Comput. Autom. Comput. Manuf. New Econ. Growth, ICAC 2015*, 2015, doi: 10.1109/IConAC.2015.7313996.

DOI:http://doi.org/10.24086/cocos2024/paper.1561