

Designing A Model of the Gravity Algorithm and Genetic Algorithm to Solve the Fuzzy Job Shop Machine Scheduling Problem in the Case of Bi-Objectives (Case Study)

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Abstract—This study presented an approach and framework for building a hybrid model of the gravity search algorithm and the genetic algorithm, to solve any of the fuzzy workshop scheduling problems (FJSSP) by fuzzing the processing times with a fuzzy number and fuzzing the due date with a fuzzy number. The gravity search algorithm was used to improve the performance of the genetic algorithm from During the generation of an initial generation of size 1, representing close-to-optimal solutions, it is used by the genetic algorithm to perform the process of mating, substitution and mutation.

The study was applied to Al-Saima Printing and Publishing Co. Ltd., where the fuzzy processing times and fuzzy due dates were recorded for four different works processed by eleven machines according to the nature of the work, and based on historical data in the company's records. Finally, the study was able to reach a set of conclusions, the most important of which is achieving the research hypothesis, which is that the hybrid model proposed by the researcher is better in obtaining the optimal sequence of actions; To reduce completion time and reach customer satisfaction by delivering the product by the specified due date using the gravity search algorithm fuzzing method and the genetic algorithm fuzzing method.

Keyword—Gravitational Search Algorithm, Genetic Algorithm, Fuzzy Set.

1- Study Methodology

The study methodology aims to deepen the vision of the intellectual and practical framework of the study by building a clear picture of its problem, objectives, importance and approach, it was presented According to the following direction:

2- Problem Statement

An industrial facility that operates on a customer-demand system struggles with how to organize a sequence Jobs according to each machine, which may lead to an increase in the total manufacturing time necessary to complete all work or may lead to an increase in wasted time for all machines or failure to meet customer requests on the due date stated in the contract concluded with the facility, Therefore, it may lead to a loss or penalty for the facility because of its significant impact on increasing the cost and lower production. Therefore, the problem of the study lies in

how to find the optimal sequence of actions (Jobs) which reduces completion time, maximizes the level of resource utilization, and achieves satisfaction The customer delivers the product on time.

- **Objectives of the study**

Scheduling plays an important role in the decision-making process in most manufacturing and production systems, and it is a very necessary condition for the stability and growth of this industry. Therefore, the study aims to:

- ❖ Presenting an approach and framework for building a hybrid model using Gravity Algorithm (GSA) and Genetic Algorithm, to solve any Fuzzy job shop Scheduling Problem (FJSSP).
- ❖ Prepare the best possible sequence of work orders that contribute significantly to decision making Appropriations that aim to achieve the general objectives of the facility by reaching customer satisfaction by delivering the product on time, reducing the overall completion time, and maximizing the level of Resource utilization, and other goals.
- ❖ Providing optimal scheduling for Al-Sayma Printing and Publishing Company Limited, aiming to achieve satisfaction The customer by meeting the specified dates for delivering the product according to the importance of the work.

3- Importance of study

- ❖ As a result of the increased intensity of competition in the markets, which results in a race against time with a goal Optimal exploitation of resources and time; Therefore, the

importance of the study emerged by presenting a model Realistic and effective, it provides operations managers with a processing tool that helps in making decisions It is appropriate to organize the sequence of production processes in a more objective manner, without taking any action Decisions based on personal estimates and personal experience.

- ❖ The practical importance of the study lies in the fact that it is a modest scientific contribution represented by an application A technique for solving fuzzy workshop scheduling problems in the Iraqi environment, as well as being A serious attempt to apply a new method that combines the tools of the gravity algorithm; To provide a hybrid system that takes advantage of the advantages of each system and exceeds its disadvantages, it can be used in environments Arabic and foreign.

4- Theoretical framework

4.1 Fuzzy Job Shop Scheduling Problem (FJSSP)

The workshop scheduling problem is one of the most complex problems in the field of operations scheduling Production, as its complexity increases with the increase in the number of jobs and the number of machines, is described as n-scheduling of jobs (jobs) on m machines (machine) and each job goes through a series of Machines: Each machine has a predetermined processing time, and it is not possible to process more than one work per machine the same machine at the same time. The workshop scheduling problem aims to reduce makespen (Reducing the time to complete work), and the idea behind this goal is to save resources (workers,

Machinery) to perform alternative tasks, or to deliver the product within the specified due date is a standard to the extent that the work is completed within the due date, because delay may lead to penalties Delay or loss of customer, and other criteria. (JSSP:)] passes through two types of restrictions [1]:

a) Sequence constraints

Also called precedence constraints, this constraint indicates that process *j* cannot process job *i* until the process that precedes it is processed, meaning that the start time (start time) of process *j* is greater than or equal to the process completion time (*j* - 1), as shown in the equation Below [2]:

$$S_{ijl} - S_{i(j-1)k} - P_{i(j-1)k} \geq 0, \forall l \neq k \dots (1)$$

Such that:

S_{ijl} =Start time for work *i* for operation *j* on machine *l*.

$S_{i(j-1)k}$ = Start time for work *i* of process (*j* - 1) on machine *k*.

$P_{i(j-1)k}$ = Processing time for work *i* of process (*j* - 1) on machine *k*.

b) Resource constraints

Job start times are the decision variables for the workshop scheduling problem that play an important role in finding the optimal sequence that achieves the facility's goals, as several jobs compete at the same time for resources (machine) [2]. Resource constraints can be explained with the equation below:

$$S_{ijk} - S_{kpk} - P_{lpk} \geq 0$$

OR ... (2)

$$S_{kpk} - S_{ijk} - P_{ijk} \geq 0$$

The figure below shows the sequence of work in the workshop scheduling problem.

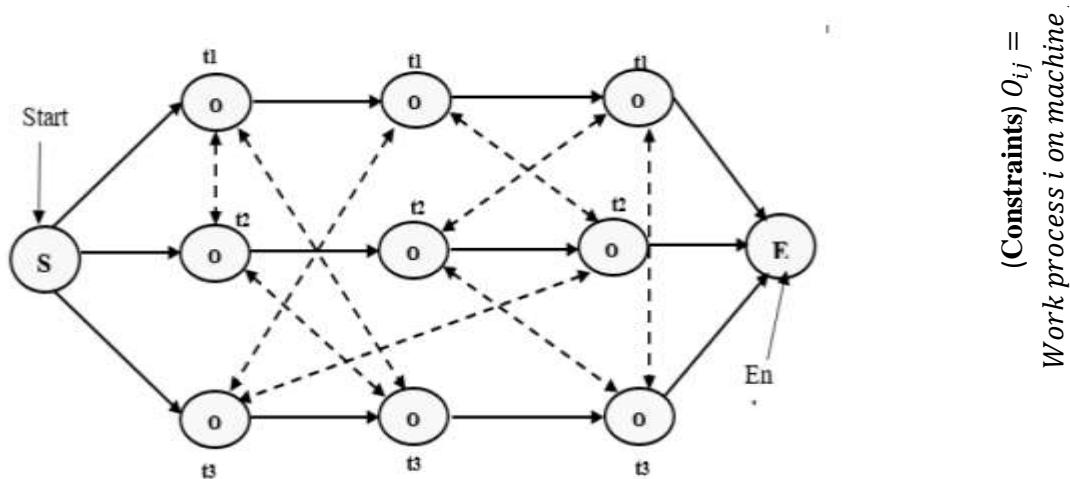


Figure (1): Represents the sequence constraints and resource constraints for the workshop scheduling problem [3]

Although many methods are used to find good solutions to the scheduling problem Workshops (JSSP), but most of the solutions and this is consistent with the nature of the problem, as most times

Processing is inaccurate due to differences in the experience of the workers and the performance of the machine, so the dates are different Eligibility is also uncertain due to the accuracy of processing times as well as the momentum and impact Experience in setting delivery dates [4] and [5]. Through this study we will discuss the use A hybrid approach to solving FJSSP because it is more realistic and accurate than the workshop scheduling problem (JSSP), where

are ineffective when applied in practice due to lack of Verify the actual processing times,

processing times will be fuzzy with triangular fuzzy numbers $(t_{ij}^1, t_{ij}^2, t_{ij}^3)$, were t_{ij}^1 Represents optimistic processing time, t_{ij}^2 Most likely processing time and t_{ij}^3 Pessimistic processing time, Fuzzing the due date with binary fuzzy numbers (d_i^1, d_i^2) such that d_i^1 Minimum due date and d_i^2 Highest maturity date ([2], [6] , [7]) As shown in Figure (2):

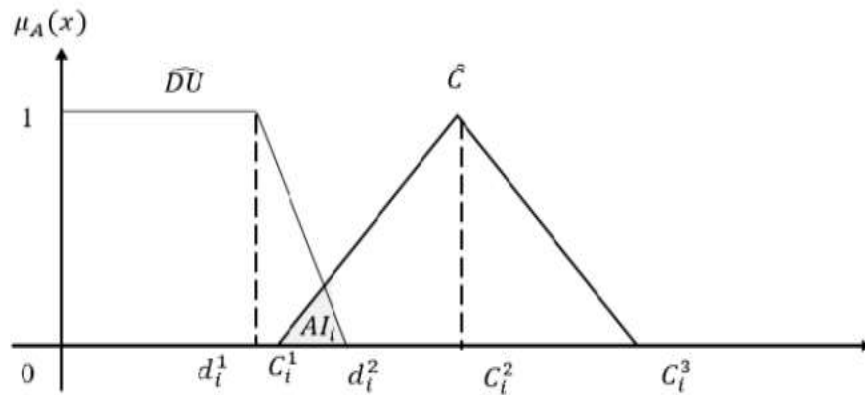


Figure (2): Fuzzy processing times and due dates

Such that (C_i^1, C_i^2, C_i^3) Triangular fuzzy numbers represent the completion time, and calculate an index Agreement index (AI) according to the equation below:

The AI value ranges between zero and one.

$$AI_i = \frac{\text{area}(\hat{C}_i \cap \widehat{DU}_i)}{\text{area}(\widehat{DU}_i)} \quad \forall i \in \{1, 2, \dots, n\}$$

S.T.

$AI_i =$ Compatibility index for work i .

$\hat{C}_i =$ Fuzzy completion time for work i .

$\widehat{DU}_i =$ Fuzzy due date for work i .

5- Gravitational Search Algorithm (GSA)

The Gravitational Optimization Technique (GSA) was suggested by [8] to address problems with effectiveness and [9]. The wider public's heuristic technique depends on gravity law and mass relationships. Agents, or solutions in the GSA community s , connect to other agents through gravity.

Every agent's effectiveness in the community was evaluated using its mass. The most suitable response has historically been the one with the higher mass.

The force of gravity is a force. Each neutrino attracting other neutrinos, and the gravitational force between both of them is inversely proportional to the distance, R, between them and directly relates to the standard deviation of their masses. Granules are merely a subset of their overall community and are inversely proportional to the distance between it, according to the law of gravitation. The law of motion states that every mass's current velocity is exactly the product of its proportionate prior velocity and its fluctuating acceleration. The rate of acceleration or alteration in the velocity of any substance is equal to the force applied to it multiplied by its mass.

The masses of the things obey gravity in a few things ways:

$$L_{ij} = G \frac{M_{aj} \times M_{pj}}{R^2} \dots\dots(4)$$

wherein G stands for the newton a constant, M_{pj} for the speed of the mirror's quantity, F for the gravitational force, with R for the separation among the two particles M_{aj} , M_{pj} .

$$a_i = \frac{L_{ij}}{M_{ii}} \dots(5)$$

Where M_{ii} is inertia mass of agent i ,

Agents are going to alter the equation's expressions for velocity V_i and location X_i throughout the investigation, so 0.

$$V_i^d(t + 1) = rand_i \times V_i^d(t) + a_i(t) \dots(6)$$

$$X_i^d(t + 1) = X_i^d(t) + V_i^d(t + 1) \dots(7)$$

A uniform random variable in the range [0, 1] is called $rand_i$ while "d" is the total number of parameters.

6- Genetic Algorithm (GA)

Genetic algorithms (GA) are heuristic-inspired adaptive search algorithms from evolutionary ideas from natural selection and natural genetics to improving target functions Very complex [10]. Holland is the first to lay the foundation for the algorithm Genetics through a series of research extending from the year 1962 AD to 1975 AD through Providing algorithms to simulate some natural phenomena through which I use concepts Reproduction, mating, and mutation, without. The term genetic algorithm is used [2]. The genetic algorithm goes through a series of Operations are as described below:

a) Chromosome Representation

It is an expression of the various variables whose values can be found or represent the available solutions that are excluded. It is also known as a symbolic string (string) and consists of a set of evidence (genes). The chromosome algorithm requires keyword analysis, and after that there are several Methods for encryption [11].

b) Fitness function

A function that measures the individual's suitability to the local environment or the proximity of the chromosome (solution) to the optimal solutions, as the solutions have a high

efficiency value. They have a greater chance of entering into the reproductive process [12].

c) Election

To preserve the most suitable chromosomes or those closest to the optimal solutions, a percentage of chromosomes is selected or selected based on the efficiency value to ensure their survival in the subsequent generation.

d) Selection

The process of selecting a pair of chromosomes as parents to carry out the

mating process and generate a new generation that carries common characteristics of the parents, and the mating process plays a role. An important role in the development of genetic algorithm [12].

e) Crossover

It is also called (the process of crossing over or exchange). It is the process of exchanging characteristics between the parents to generate two new children (two new chromosomes) that carry characteristics common to the parents. As shown in the figure below:

First parent	0000000000	First son	1110000000
Second parent	1111111111	Second son	0001111111

Table (1) shows the operation of crossover According to the genetic algorithm approach

It is the process of replacing one or more genes that are randomly selected among new children generated through the process of crossover, as shown in the following figure:

f) Mutation

First son	111000 0 000	The first son after the mutation	111000 1 000
Second son	000111 1 111	The Second son after the mutation	000111 0 111

Table (2) Explains the mutation process According to the genetic algorithm approach

g) Stop Criterion

It is a measure or criterion used to stop Genetic algorithm is about generating new solutions or generations and there are different methods to determine when The execution of the genetic algorithm stops, and these methods include [12]:

1. Reaching the maximum specified number of repetitions.
2. Members of a population are identical for more than one cycle (replication).
3. The genetic algorithm has the best individual for more than one iteration.
4. If the fitness function is reached within a specific range of Before the user.

7- Model parameters

➤ **Minimum Index Agreement**

$$MinAI = \min(AI_i) \text{ where } i \in \{1, 2, \dots, n\} \dots (8)$$

➤ **Average Index Agreement**

$$AAI = \frac{\sum_{i=1}^N A_i}{N}, i \in \{1, 2, \dots, n\}$$

..... (9)

➤ **Satisfaction grade of weighted tardiness of jobs**

$$WAI = \sum_{i=1}^n W_i A_i, i \in \{1, 2, \dots, n\}, W_i = [0, 1]$$

..... (10)

Such that

W_i represents the importance of the work weighted according to the delay penalty or according to the importance of the customer.

➤ **Satisfaction grade of number of tarty jobs**

$$NT_i = \begin{cases} 1 & \text{if } A_i \leq \lambda \\ 0 & \text{if } A_i > \lambda \end{cases}, i \in \{1, 2, \dots, n\}, \lambda = (0.5, 1) \dots (11)$$

$$NTAI = \sum_{i=1}^n \frac{NT_i}{n}$$

.... (12)

➤ **Degree of satisfaction with all criteria**

$$Obj - fun = \frac{(\min AI + AAI + WAI + NTAI)}{4} \dots$$

(13)

The goal of the above criteria is to maximize the degree of satisfaction.

8- Practical framework

• **For case study:**

The case study is considered an effective educational tool in many

The actual processing times for the four tasks, and the weighted importance of each, were also recorded Work, as shown in Table (3) and (4) respectively.

disciplines. For what you offer There are many benefits for the researcher, such as bridging the gap between theory and practice, and between academic aspects Workplaces also provide an opportunity to define the problem and understand and clarify situations and evaluation of courses of action. Due to the importance of the case study and the above, we will discuss an overview of the company Al-Sayma Printing and Publishing Limited, and the data that was recorded, in addition to the method Used by the company for scheduling.

• **Recorded data:**

For the purpose of testing the model proposed by the researcher, we made a field visit to a company Al-Symaa Printing and Publishing Limited for the period from (23/1/1023 until 11/3/2023), as the vague processing times were recorded for four different jobs processed by eleven machines according to the nature of the job (job), based on the historical data found in the company's records for a period of one year and the experience of specialists, as well as. Shown in Table (3)

- ❖ Work 1 Print a book of 500 copies.
- ❖ Work 1 wall calendar 1000 copies.
- ❖ Work 9 carton box of 1000 copies.
- ❖ Work 4 Receipt for 10 books.

Table (3) represents the fuzzy processing time for Al Ghadeer Company
(Source: Relying on company records)

The Job	Fuzzy processing time in hours for each operation								Fuzzy due date
	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	
Work1	(16,24,40) M1	(8,12,16) M2	(14,68,24) M3	(8,12,16) M4	(4,6,8) M5	(8,12,16) M6	(4,6,8) M7	(3,5,7) M8	(120,160)
Work2	(1,2,3) M1	(1,2,3) M2	(0,5,1,2) M3	(0,5,1,1,5) M8	(2,3,4) M9	0	0	0	(24,32)
Work3	(8,16,24) M1	(2,3,4) M2	(2,4,5) M3	(2,4,6) M9	(8,16,24)	0	0	0	(72,96)
Work4	(0,5,1,4) M1	(0,5,1,1,5) M2	(2,3,4) M3	(1,2,3) M6	(0,5,1,1,5) M11	(0,5,1,1,5) M8	0	0	(32,40)

Table (4): Actual processing times (Source: Based on actual recorded processing times)

The Job	Actual processing time in hours								Completion time in hours
	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	
Work1	22 M1	13 M2	17 M3	10 M4	6 M5	12 M6	5.5 M7	5 M8	(120,160)
Work2	2 M1	2 M2	1.5 M3	1.5 M4	3 M5	0	0	0	(24,32)
Work3	17 M1	3 M2	4 M3	5 M4	12 M5	0	0	0	(72,96)
Work4	1 M1	1 M2	3.5 M3	2 M4	1.5 M5	1 M6	0	0	(32,40)

Table (5) The relative importance of work (Source: Al-Syama Printing and Publishing Company Limited)

The Job	Weigh the importance of the work
Work1	0.4
Work2	0.2
Work3	0.2
Work4	0.2

• **Scheduling used by the company:**

The scheduling used by the company depends on the due date, importance of the work and volume Work, through the data recorded in Table (3), we note that Work 1 and Work 4 do not require a long time to process, unlike Work 1, which requires a long

time to process. Through the interview with the authorized director, it was shown that work that does not require a long time is processed. Either at the beginning or before the due date, within the period that the work needs to process it according to momentum

Work, so work 4 was processed first, then work 2, and then work 1. This is because the process was processed the first work 3 requires time (8, 16, 24) as shown in Table

(3) and may affect the completion time of Work 1. Table No. (4) shows the scheduling used by Company.

Table (6) Scheduling Al-Sayma Printing and Bulletin Company Limited (Source: Relying on the schedule used by the company)

Machines										
M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
4	4	4	1	1	4	1	2	2	3	4
2	2	2	0	0	1	0	4	3		0
1	1	1	0	0	0	0	1	0	0	0
3	3	3	0	0	0	0	0	0	0	0

9- Building the model

9.1 Form inputs

FT: A matrix of degree $(3m \times n)$, where m represents the number of machines and n The number of jobs, and the elements of the matrix represent the fuzzy processing times of the process (i, j) .

$$\begin{matrix}
 J_i & O_1 & O_2 & \dots & O_m \\
 1 & \widehat{t_{1,1,k}} & \widehat{t_{1,2,k}} & \dots & \widehat{t_{1,m,k}} \\
 2 & \widehat{t_{2,1,k}} & \widehat{t_{2,2,k}} & \dots & \widehat{t_{2,m,k}} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 n & \widehat{t_{n,1,k}} & \widehat{t_{n,2,k}} & \dots & \widehat{t_{n,m,k}}
 \end{matrix}
 , k = 1, 2, \dots, m$$

Such that:

$\widehat{t_{i,j,k}}$ = A score matrix (1×3) representing the fuzzy processing time for each process.

M: A matrix of degree $(n \times m)$. The elements of the matrix represent the rank of the process (Operation) for each machine k (Machine) according to the job i (Job).

$$\begin{matrix}
 J_i & M_1 & M_2 & \dots & M_m \\
 1 & O_{1,1} & O_{1,2} & \dots & O_{1,m} \\
 2 & O_{2,1} & O_{2,2} & \dots & O_{2,m} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 n & O_{n,1} & O_{n,2} & \dots & O_{n,m}
 \end{matrix}$$

FDUE: A score matrix $(2 \times n)$ representing the fuzzy due date for each action (Job).

$$\begin{matrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{matrix} \begin{bmatrix} \widehat{DU}_1 \\ \widehat{DU}_2 \\ \vdots \\ \widehat{DU}_n \end{bmatrix}$$

Such that:

\widehat{DU}_i = A matrix of degree (1 × 2) representing the fuzzy due date of job *i*.

WJ: A vector of degree (1 × *n*) representing the relative importance of the business.

P: The size of the initial population to be generated.

N: The number of generations to be generated (number of iterations).

Ima: The acceptable compatibility index value for each work, and its value ranges from (0.5.1).

9.2 Model outputs

S Fuzzy: A matrix of degree (3*m* × *n*) representing the fuzzy start time for each operation.

C fuzzy: A matrix of degree (9*xn*) representing the fuzzy completion time for each work.

$$\frac{A^1+A^2+A^3}{4}$$

9.3 Objective function

- ❖ $maxZ_1 = minAli$
- ❖ $maxZ_2 = AAI$
- ❖ $maxZ_3 = wAI$
- ❖ $maxZ_4 = NTAI$
- ❖ $maxZ_5 = opt - fun$

I. Steps for using a genetic algorithm to address the problem of fuzzy workshop scheduling

Joq _Seq: A matrix of degrees (*m* × *n*) representing the sequence of jobs (jobs) for each machine.

AI: A matrix of score (1 × *n*) representing the value of the compatibility index for each job, as

Explained by equation (3).

min AI: represents the lowest value of the Agreement Index Explained by equation (4).

AAI: represents the average agreement index (explained by the equation).

WAI: The value of the agreement index represents the relative importance of each job (job), as

shown in equation (6).

NTAI: Degree of satisfaction with the number of late works, as shown in equation (7,9).

opt _ fun: the degree of satisfaction with all criteria, as shown in equation (3).

E(Ci): The expected completion time for each job.

.... (14)

1- Generate initial solutions (P) from times and calculate the completion time (Completion Time) for each job and the fitness function. The function will be used Below is an efficiency function in the current study:

$$Fit = \max \left(\frac{C_i^3}{DU_i^2} \right), \forall i = \{1,2, \dots, n\} \quad \dots (15)$$

Such that:

C_i^3 = Pessimistic completion time for work i .

$$DU_i^2 = \text{Greatest due date for work } i.$$

- 2- **Elitism:** The best 50% are selected according to the efficiency function from the parental generation to ensure their continuity in the next generation.

$$El = px \ 0.5 \quad \dots (16)$$

- 3- **Selection:** This stage is summed up by selecting the appropriate pairs to carry out the mating process. The tournament selection method will be used, which consists of selecting 16 individuals randomly from the community. Then the match process will be conducted according to the efficiency function and the best two individuals will be selected for the mating.

$$G = \frac{p-El}{2} \quad \dots (19)$$

- 4- **Crossover:** This process consists of exchanging chromosomes between the parents. The arithmetic crossover method will be used according to the following equation:

$$Child_1 = a \times parent_1 + (1 - a) \times parent_2$$

$$Child_2 = a \times parent_2 + (1 - a) \times parent_1 \quad \dots (17)$$

The value of a is a random number ranging between (0,1).

- 5- **Mutation:** 0.02 of the number of genes per chromosome is selected to perform Mutation process.

$$Mu = (m \times n) \times 0.02 \quad \dots (18)$$

- 6- The process is repeated (4,5,6) times to generate new children and calculate the fitness function for them.

- 7- Replacing the new generation that was generated in place of the worse ($G*2$) of the previous generation.
- 8- The process is repeated N times starting from step (3).

II. Steps to make the Gravity Algorithm (GSA) and its training mechanism.

- 1- Calculating initial start times (start time according to the sequence of operations for each work only).

$$S_{ijk} = S_i(j - 1)k + T_i(j - 1)k ; i = 1, \dots, n, j = 2, \dots, m, k = 1, \dots, m \quad \dots (20)$$

- 2- Finding the variable indicator layer (Position) based on the start times, as shown below:

$$y_{ipk} = 1 \text{ if } S_{ijk} \leq S_{pik} \forall p > i, l \neq i$$

$$y_{ipk} = 0 \text{ if } S_{ijk} > S_{pik} \forall p > i, l \neq i$$

$$\dots (21)$$

- 3- Resource constraints, symbolized by the symbol RC, this layer consists of ($b \times 2$) locations, as shown in the figure below.

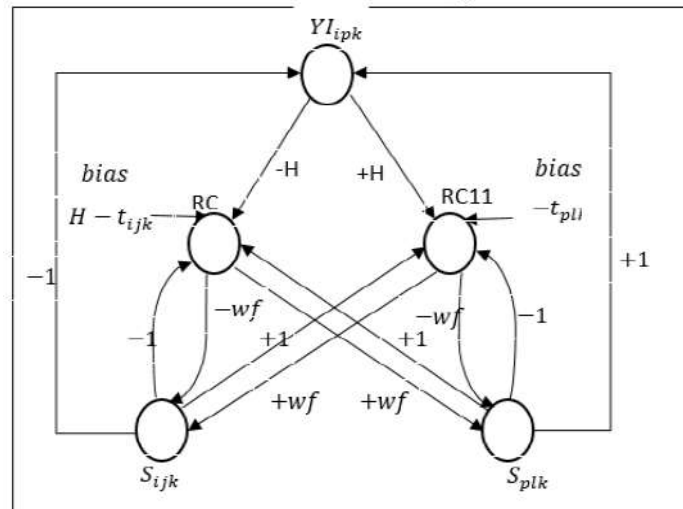


Figure (3) RC layer resource constraints algorithm Source [1]:

- 4- Find the Sequence Constraints layer, symbolized by the symbol SC as shown in the figure below:

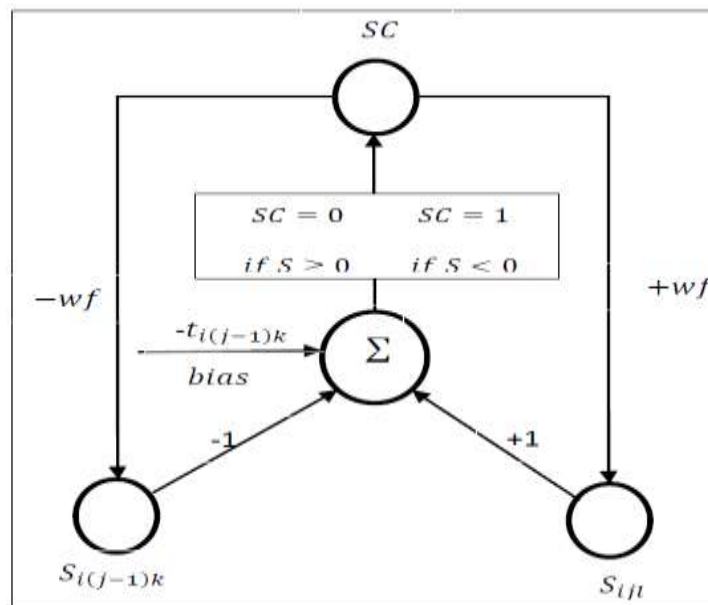


Figure (4) Sequence Constraint Algorithm (SC Layer) Source [1]:

Since:

$$NS = S_{ijl} - S_{i(j-1)k} - t_{i(j-1)k}$$

$$SC = \begin{cases} 1, & NS < 0 \\ 0, & NS \geq 0 \end{cases} \dots (22)$$

If the value of SC equals 1, an adjustment is made to the value of S (start time) through Wf reactions.

5- Repeat steps (4,3) until all values of RC and SC equal zero.

6- Training process.

- Calculating the completion time for each work.

$$C^3 = S^3(:, \text{end}) + T^3(:, \text{end})$$

.... (23)

Since:

C^3 = A vector of degree $(1 \times n)$ representing the pessimistic completion time for each job.

$S^3(:, \text{end})$ = A vector of degree $(1 \times n)$ representing the latest start time for each job.

$T^3(:, \text{end})$ = A vector of degree $(1 \times n)$ representing the last processing time for each

job.

- Calculate the Stop Index (SI), according to the equation below:

$$SI = \frac{C_i^3}{DU_i^2}; i = \{1, 2, \dots, n\}$$

..... (24)

Since:

DU_i^2 = Highest due date for work i.

- If the value of **Max(SI)** is less than or equal to 1, we stop, meaning the solution has been reached. Optimum, otherwise make an adjustment to the weights to change the sequence of actions.
- Return to step (3). The process is repeated 5000 times or the optimal solution is reached.

III. Hybrid system (genetic algorithm and gravity):

- 1- Generate the location layer randomly a P number of times.
- 2- Find solutions for each layer generated using gravity search, as described in paragraph (II).
- 3- Calculate the fitness function for each solution, and arrange it in ascending order
- 4- Executing the genetic algorithm described in paragraph (I). (Starting from step) 1

9.4 Model test:

It is not accurate to accept or reject a particular hypothesis unless there is a way to prove it Whether it is true or false, therefore testing the hypothesis is one of the important matters in research. Scientific evidence to prove its validity. In this section, we will discuss testing the model by comparing the model. The proposal with the method used by Al-Symaa Press, in addition to comparing the model with Genetic algorithm and gravity search for each system separately by repeating the scheduling process more than once for each system and recording the results.

- Comparison of the proposed model with the scheduling method used by Al-Symaa Company:
- First: Comparison in the case of fuzzy processing time.
 - By applying the scheduling used by Al-Symaa Company, Table (6) to a specific time
 - Fuzzy processing Table (3) The results were obtained below:

Table (7): Fuzzy start time for each process according to the schedule used by the company

The Job	Fuzzy processing time for each process								Fuzzy due date
	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	
Work1	(1,5,3,5) M1	(17.5,27,45) M2	(25.5,39,61) M3	(39.5,57,85) M4	(47.5,69,101) M5	(51.5,75,109) M6	(59.5,87,125) M7	(63.5,93,133) M8	(120,160)
Work2	(0,5,1,2) M1	(1,5,3,5) M2	(3,5,8) M3	(3.5,6,10) M8	(4,7,11,5) M9	0	0	0	(24,32)
Work3	(17.5,27,45) M1	(25.5,43,69) M2	(39.5,57,85) M3	(41.5,61,90) M9	(8,16,24) M10	(4.5,8,12) M8	0	0	(72,96)
Work4	(0,0,0) M1	(0.5,1,2) M2	(1,2,3,5) M3	(3,5,7,5) M6	(3,5,7,5) M11	(0,5,1,1,5) M8	0	0	(32,40)

Table (8)P: Fuzzy completion times according to the schedule used by the company
(Source: Based on the schedule used by the company)

Fuzzy completion time			
The Jobs	Early time	Most likely time	Pessimistic time
Work1	66.5	98	140
Work 2	6	10	15.5
Work 3	51.5	81	120
Work 4	5	9	13.5

Table (7) Compatibility index according to company scheduling

Compatibility index	
The Jobs	Jobs
Work1	1
Work 2	1
Work 3	0.5
Work 4	1

By applying the proposed model to the problem shown in Table (3), where an initial generation of size (P=30) was generated, and the number of generations to be generated was (N=500). The following results were obtained:

Table (9) Fuzzy start time according to the proposed model (Source: Based on the results of the proposed model.)

The Jobs	Fuzzy start time for each process								Fuzzy due date
	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	
Work1	(9,19,29) M1	(25,5,23,69) M2	(33,55,85) M3	(47.5,73,109) M4	(55.5,85,125) M5	(59.5,91,133) M6	(67.5,103,149) M7	(71.5,109,157) M8	(120,160)
Work2	(0.5,1,2) M1	(1.5,3,5) M2	(3,5,8) M3	(3.5,6,10) M8	(4,7,11.5) M9	0	0	0	(24,32)
Work3	(1.5,3,5) M1	(9.5,19,29) M2	(11.5,22,33) M3	(13.5,26,38) M9	(15.5,30,44)	0	0	0	(72,96)
Work4	(0,0,0) M1	(0.5,1,2) M2	(1,2,3.5) M3	(3,5,7.5) M6	(4,7,10.5) M11	(4.5,8,12) M8	0	0	(32,40)

Table (10): Tabulation for the proposed model (Source: Based on the results of the proposed model.)

Sequence of work according to each machine										
M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
4	4	4	4	0	2	2	0	4	0	0
2	2	2	0	0	0	4	0	0	0	0
3	3	3	0	3	3	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	0

Table (11): Fuzzy completion time according to the proposed model (Source: Based on the results of the proposed model.)

Fuzzy completion time		
The Jobs	Early time	Pessimistic time

Work1	74.5	114	164
Work 2	6	10	15.5
Work 3	23.5	46	68
Work 4	5	9	13.5

Table (12): Compatibility index for each work according to the proposed model (Source: Based on the results of the proposed model.)

Compatibility index	
The Jobs	Compatibility index value
Work1	0.9
Work 2	1
Work 3	1
Work 4	1

The table below shows a comparison of results between the scheduling method used by the company The method proposed by the researcher.

Table (13) comparing the results of the proposed model with the results of company scheduling

Calibrated indicator value	Calibrated indicator value	
	The proposed model	The proposed model
Compatibility index rate	0.9955	0.8851
Lowest agreement index	0.98	0.5404
Satisfaction index according to the importance of work	0.9921	0.9081
Satisfaction index with the number of late works	1	0.75
Optimal Function	0.9926	0.7

We note from the table above that the schedule was found using the model proposed before. The researcher is better than the tabulator used by Al-Symaa Printing and Publishing Company Limited according to the four criteria used (average concordance index, lowest concordance index, index. Satisfaction according to the importance of the work, and the satisfaction index according to the number of late works in the case (0.8= λ am).

Through the scheduling used by Al-Symaa Printing Press shown in Table (6) and through the actual processing times that were recorded by the researcher and the clarifier. In Table (4) we find:

Table (14) Completion time according to processing times the real and proposed scheduling model

The Job	End Time
Work1	110.5
Work 2	11.5
Work 3	44
Work 4	10
Average completion time	44

Table (15) Belonging function according to real processing times And the proposed scheduling model

The Job	The value of belonging
Work1	1
Work 2	1
Work 3	1
Work 4	1
Average completion time	1

Below is a table showing the comparison between the results obtained from the scheduling process Used by Al-Symaa Press and the results reached according to the proposed model Based on actual processing times.

Table (16) Comparison of the proposed model with the company’s scheduling based on times
Real processing

Standard	Proposed model	Company scheduling
Actual completion time rate	44	47.75
The lowest affiliation function	1	0.8333
Average belonging function	1	0.9583

We note from the table above that the average completion time for the proposed model is less than the average completion time Completion of the method used by the company in addition to all the values of the affiliation functions It equals one, and from the above we conclude that the tabulation used by the researcher is better.

9.5 Comparison of the proposed model with the genetic algorithm and the gravity search algorithm:

In this paragraph we will compare scheduling using the hybrid model using a genetic algorithm.

Gravity and scheduling of the gravity search algorithm by applying it to the two problem (6x6) and as shown in Table (17). The problem will be repeated ten times to clarify the difference in the randomness of the results between the three systems by generating (90) initial generations and the number of iterations of the new generations

that will be generated. (1000,500) generations for the first and second problem, respectively. The relative importance of the two problems is equal, meaning there is no

preference for one work over the other. The acceptable compatibility index for the first problem is ($\lambda=0.8$) and for the second problem ($\lambda=0.6$).

Table (17) The problem of grade (6 x 6)

The Job	Fuzzy processing time in hours for each operation						Fuzzy due date
	OP1	OP2	OP3	OP4	OP5	OP6	
Work1	(5,7,10) M8	(10,14,17) M5	(1,3,5) M4	(1,3,5) M3	(4,6,8) M2	(9,10,11) M1	(81,88)
Work2	(6,7,8) M7	(9,13,317) M1	(8,12,13) M3	(2,3,4) M6	(10,13,16) M4	(2,3,4) M2	(66,80)
Work3	(4,5,7) M3	(10,11,12) M1	(9,12,16) M5	(8,12,13) M2	(6,9,12) M6	(9,10,11) M1	(89,92)
Work4	(1,2,4) M4	(2,4,5) M5	(5,7,8) M6	(5,8,10) M3	(3,5,7) M1	(4,7,9) M4	(52,60)
Work5	(9,11,15) M4	(4,6,9) M1	(1,2,3) M5	(10,11,15) M6	(4,7,8) M2	(10,11,12) M3	(91,96)
Work6	(6,7,9) M5	(1,2,4) M3	(6,9,11) M2	(10,14,18) M6	(1,2,3) M4	(9,13,14) M1	(54,59)

9- Most important conclusions

1. The study reached the proposed hybrid model to solve the problem of scheduling workshops Fuzzy (FJSSP), using the gravity algorithm to improve the genetic algorithm is presented better results than using each system separately, especially as the complexity of the problem increases, as it arises Gravity search algorithm by providing initial solutions that are optimal or close to optimal helps the algorithm Genetics help you get good results quickly.
2. Al-Symaa Printing and Publishing Company Limited adopts traditional rules in scheduling work, such as: The EDD rule, LPT rule, and SPT rule, depending on experience and work requirements, are static rules. This

makes both work-in-process (WIP) inventory and total Inventory is high, as well as inaccurate delivery dates in some cases.

3. The use of fuzzy due dates and fuzzy processing times creates a workshop scheduling problem The work contributes to addressing uncertainty regarding the dates of product delivery to the customer. a result Differences in worker experience and machine performance, through the adoption of scheduling based on uncertainty Contributes to making more accurate decisions.

Future study

- We suggest applying the hybrid model, especially after proving its ability to provide a good scheduling solution The problem of scheduling

fuzzy workshops to address other problems such as the problem of flow workshop (Flow Shop Scheduling) or the problem of flexible job shop scheduling.

- Application of the hybrid model to the dynamic rescheduling problem Partially or generate a new schedule in response to a change or event occurring in the production environment Such as a machine
-

malfunction, the arrival of new work, or the cancellation of work orders.

- Conduct test studies to compare the performance of the gravity search algorithm with methods other algorithms used to address the fuzzy workshop scheduling problem e.g Bee algorithm, blocked search, pseudo-fusion, or other applications developed.

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