A BILATERAL FILTERS FOR DE-NOISING NOISY IMAGES

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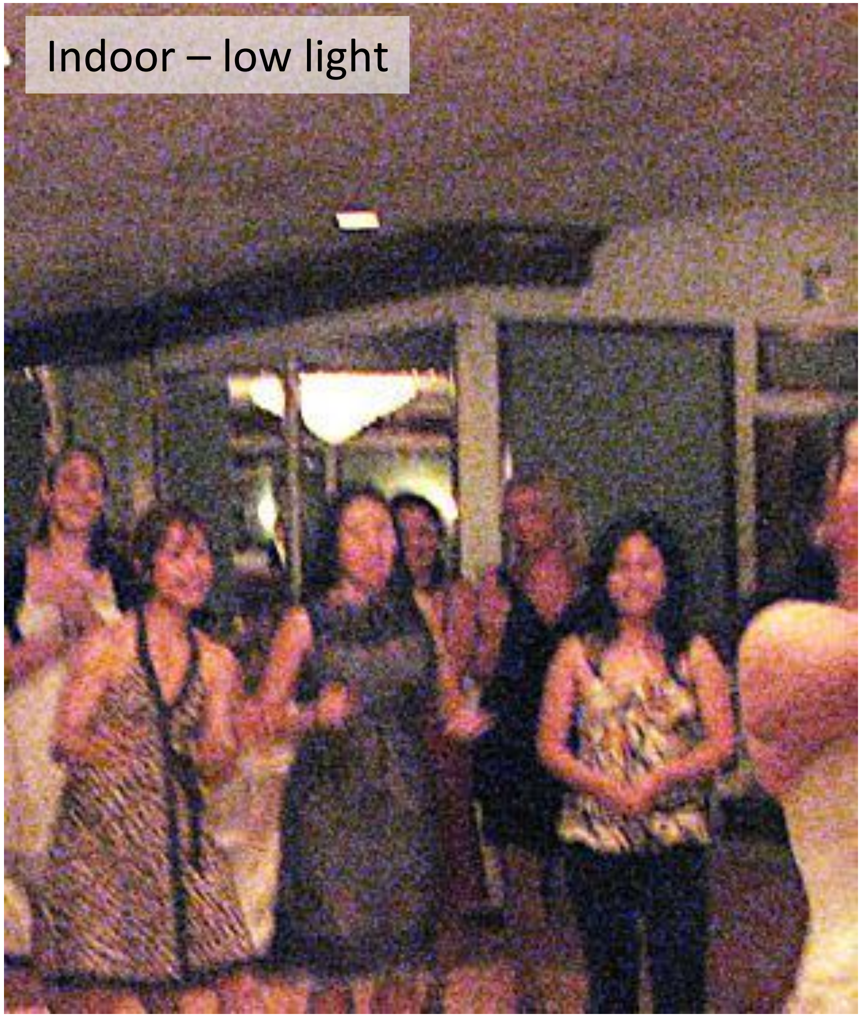
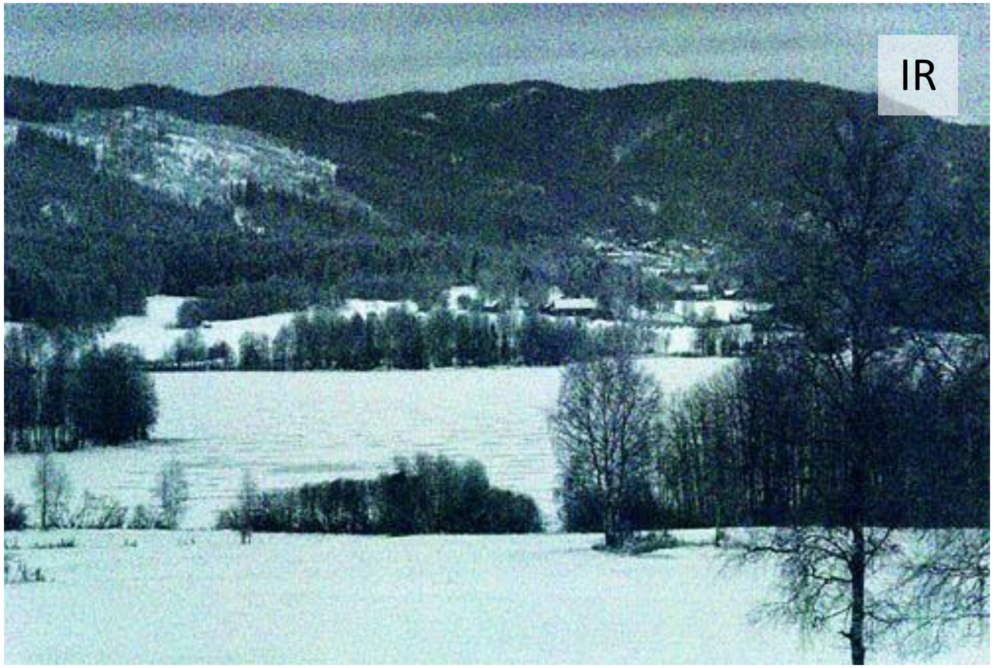
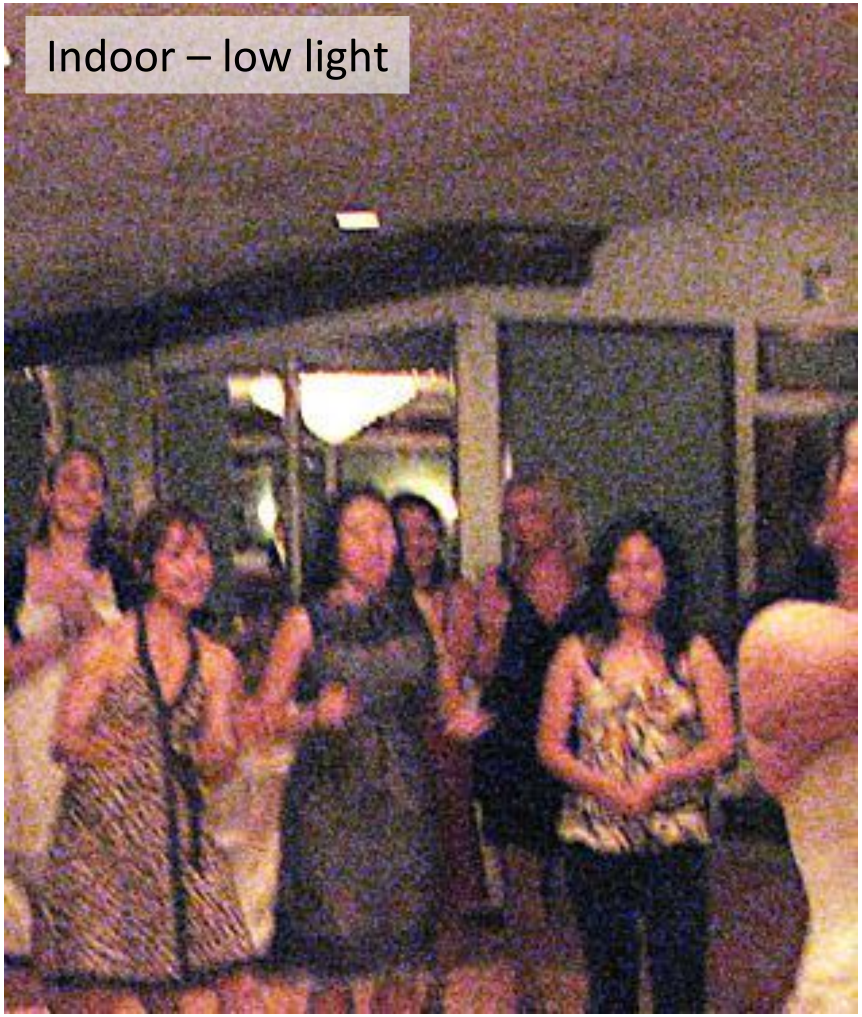
A Tour of
Image Denoising

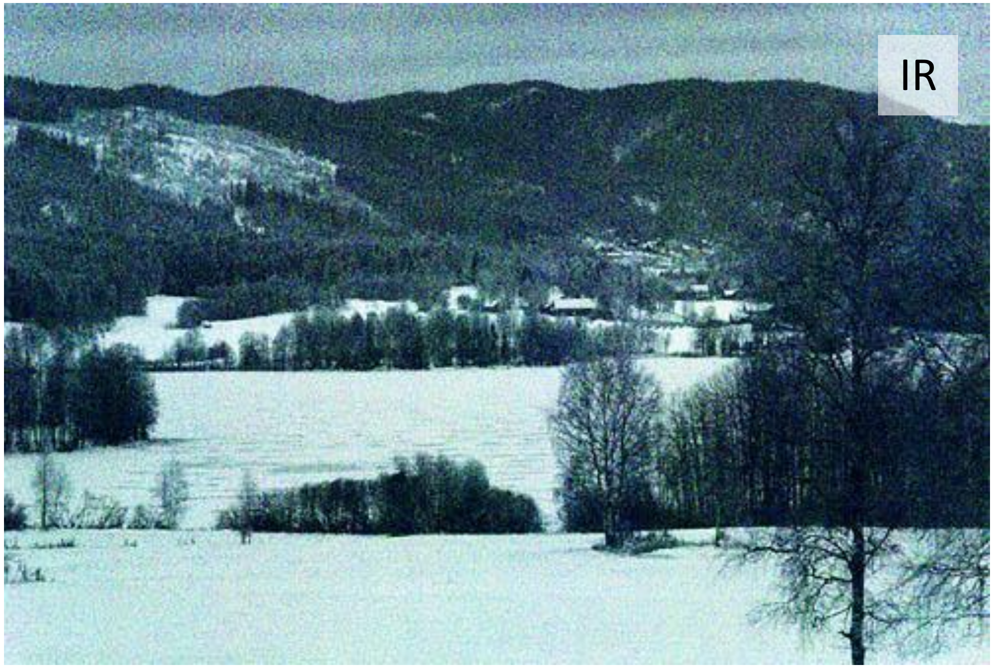

Indoor – low light


IR


US

Can we (humans) denoise?

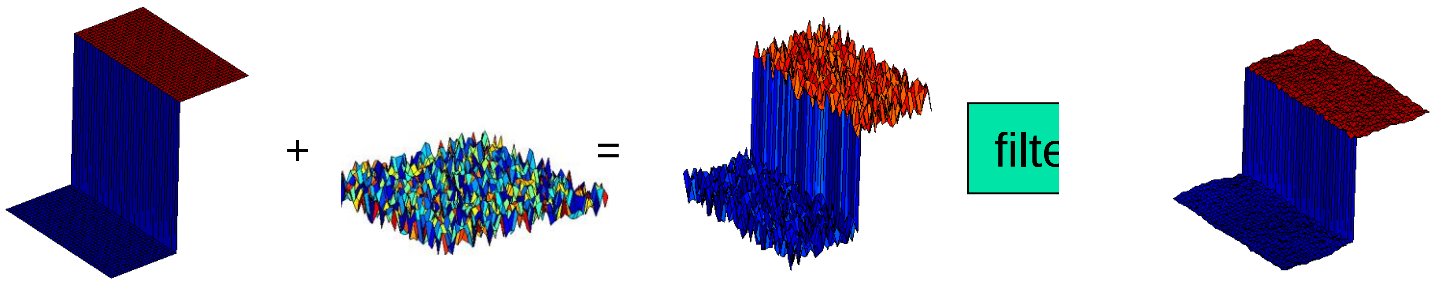

Indoor – low light


IR


US

Denoising

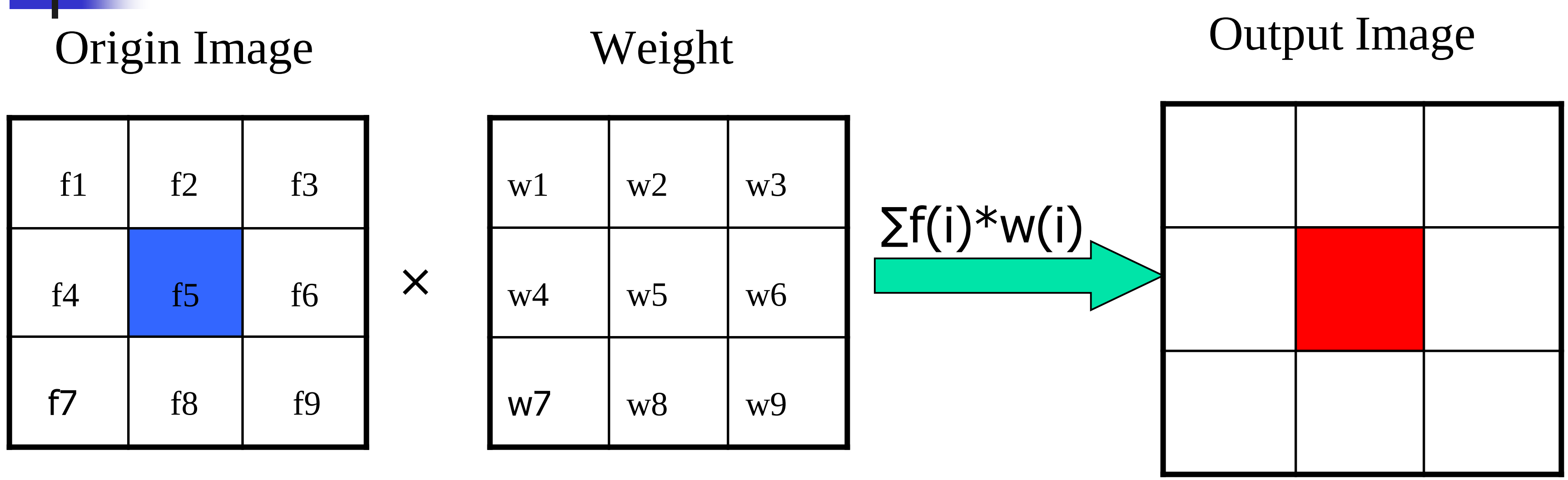
- Input (scanned) mode
  - Additive noise
- Noise free model
  - Preserve features



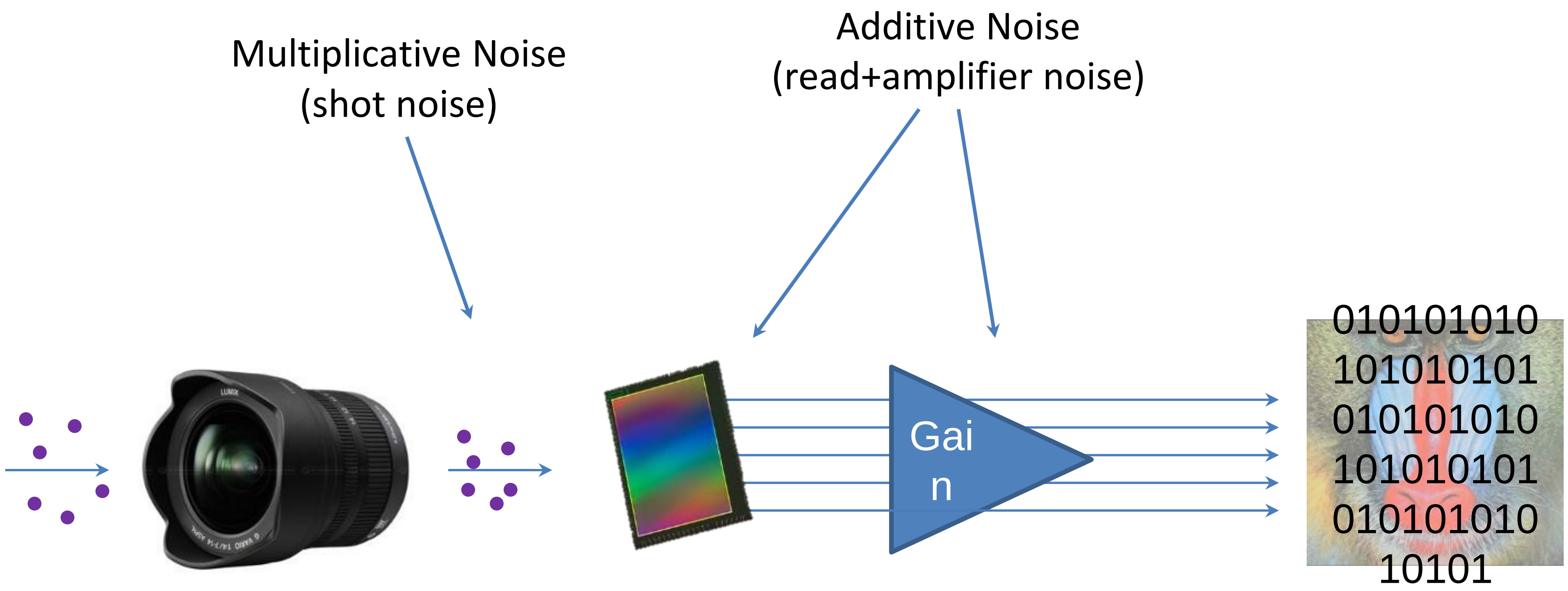
Filter Functions

- Noise removal
- Image smoothing
- Preserve features

How to Compute Every Pixel



Sources of Noise

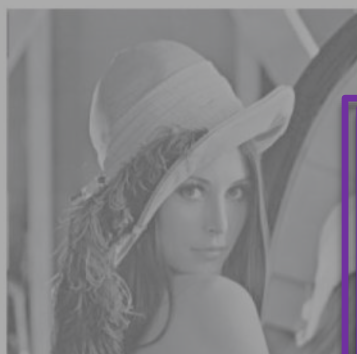


Problem Definition

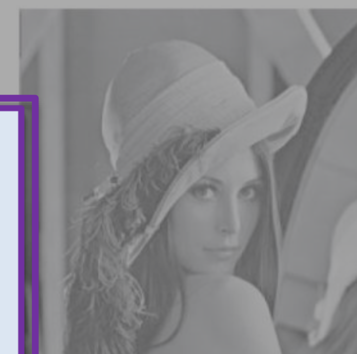
Visual Quality

MSE / PSNR

x



$\hat{x}$

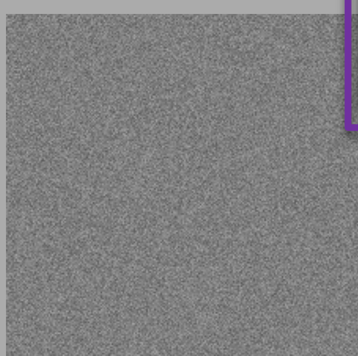


Mean Square Error

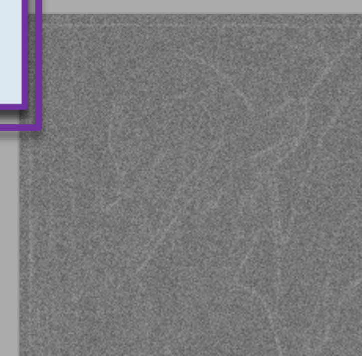
$$\text{MSE} = \|\hat{x} - x\|_2^2$$

Peak Signal-to-Noise ratio

$$\text{PSNR} = 20log_{10}\frac{255}{\sqrt{\text{MSE}}}$$



$y = x + n$



n

Noise? → $\hat{n} = y - \hat{x}$

Image Denoising

noisy image

naïve denoising
Gaussian blur

better denoising
edge-preserving filter

Smoothing an image without blurring its edges.

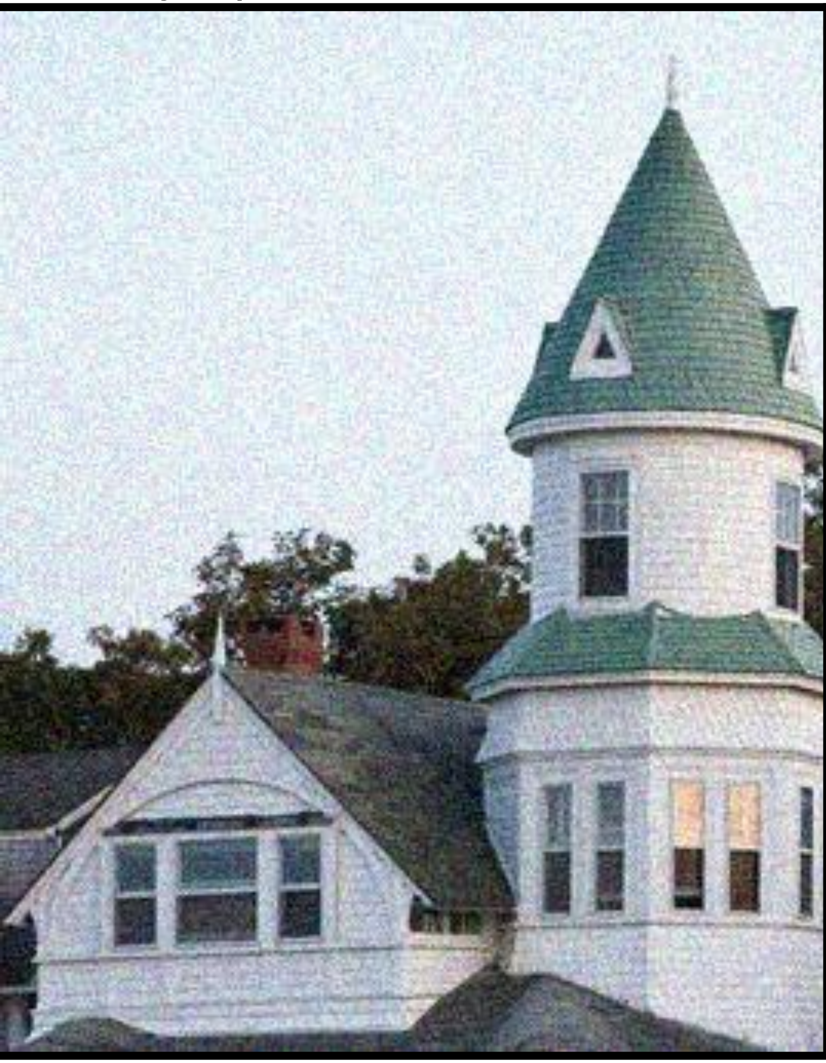
Basic denoising

Noisy input

Median 5x5

Basic denoising

Noisy input



Bilateral filter 7x7 window

Tone Mapping

[Durand 02]

HDR input

Tone Mapping

[Durand 02]

output

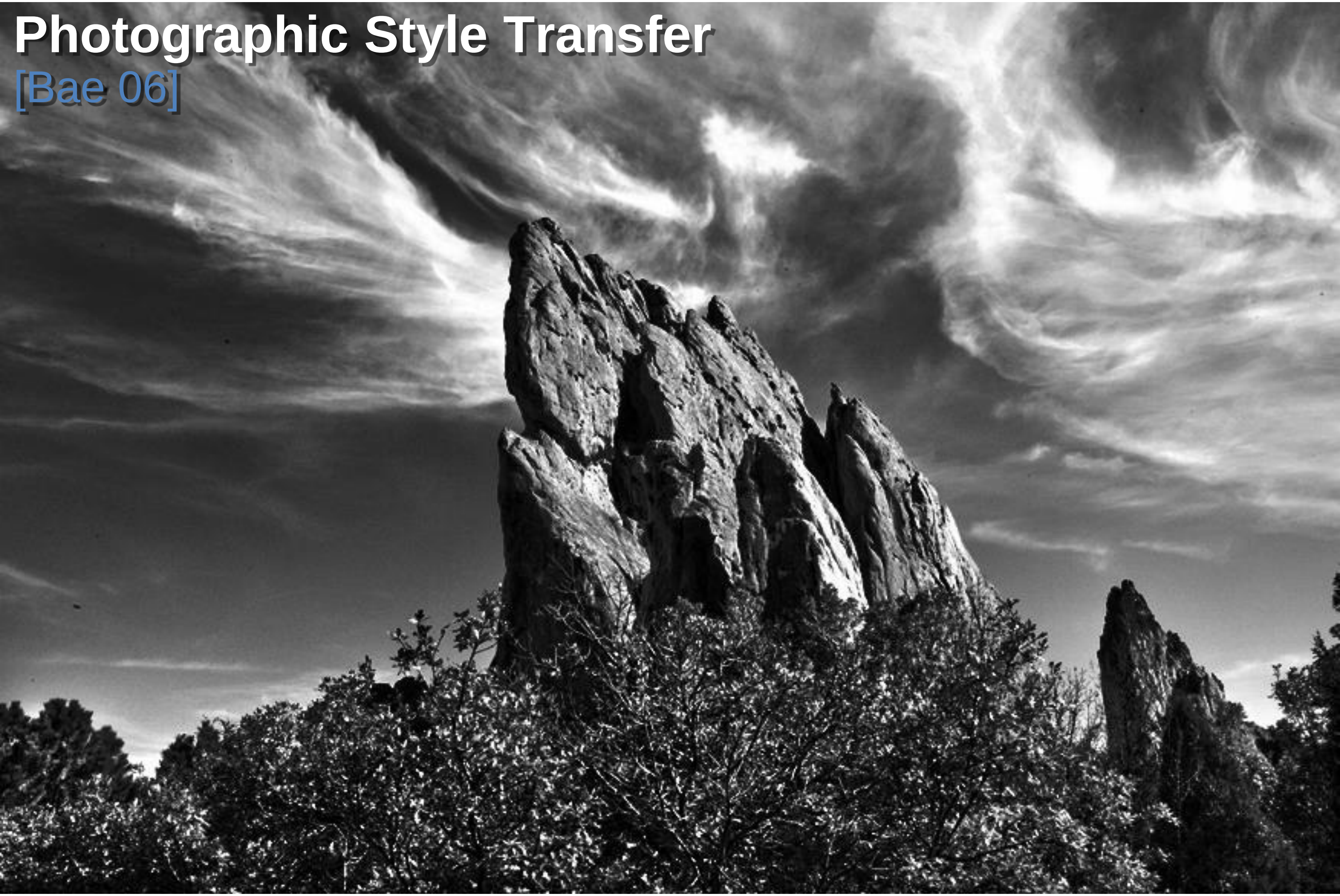
Photographic Style Transfer

[Bae 06]

input

Photographic Style Transfer

[Bae 06]



output

Cartoon Rendition

[Winnemöller 06]

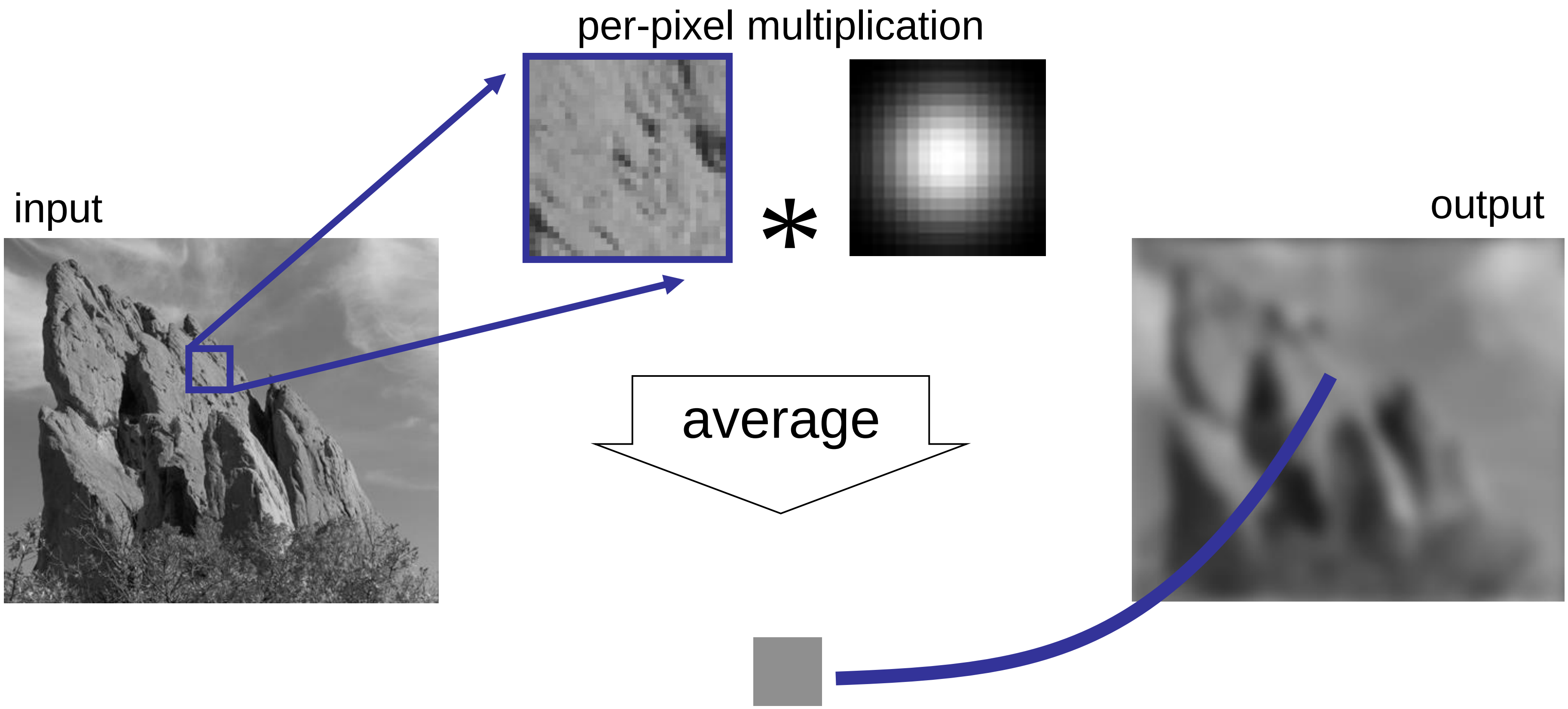
input

Cartoon Rendition

[Winnemöller 06]

output

Gaussian Blur




input


box average

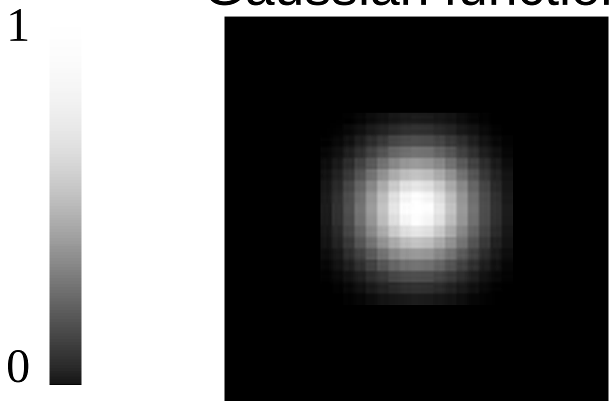

Gaussian blur

Equation of Gaussian Blur

Same idea: **weighted average of pixels**.

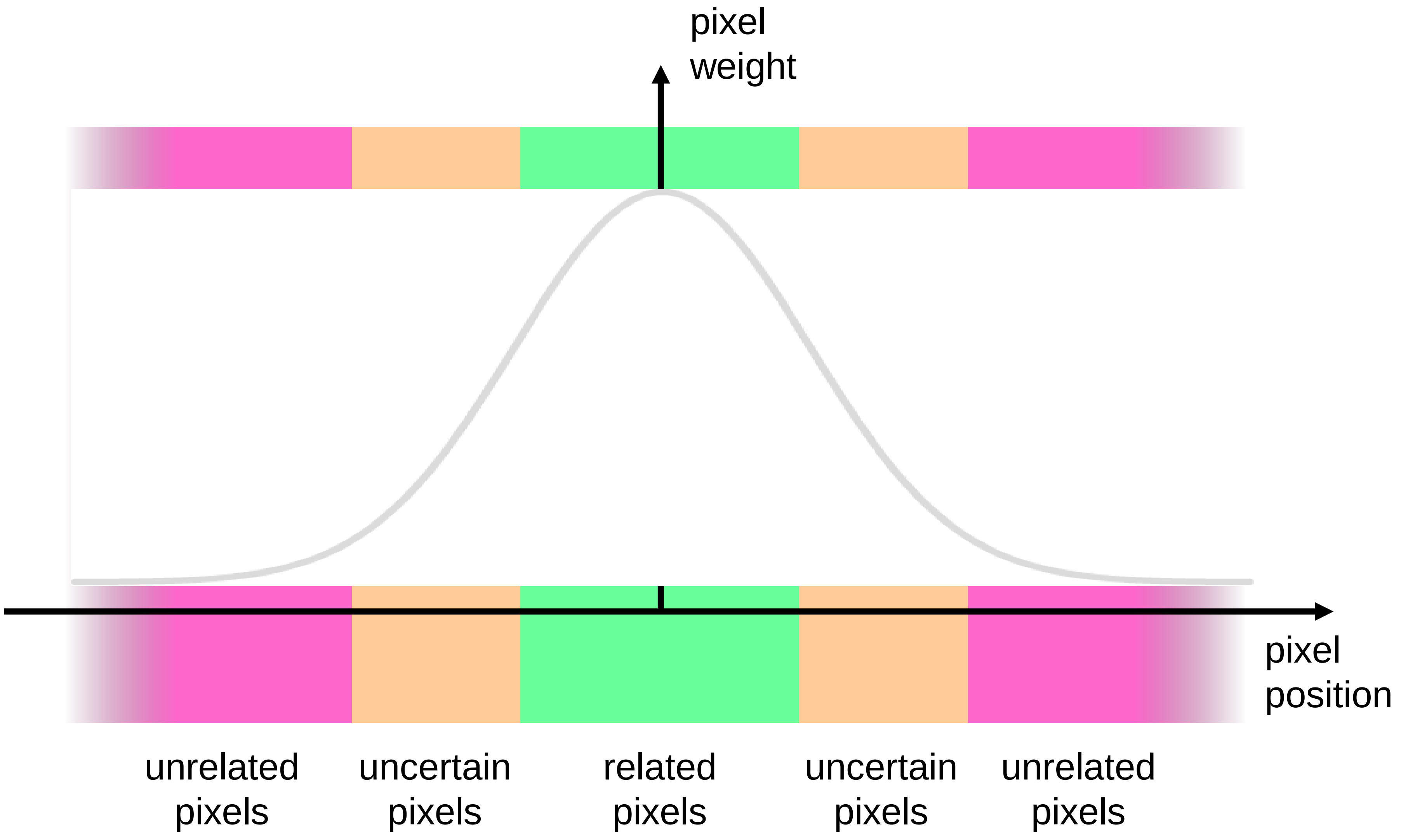
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\| \mathbf{p} - \mathbf{q} \|) I_{\mathbf{q}}$$

normalized Gaussian function



Gaussian Profile

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

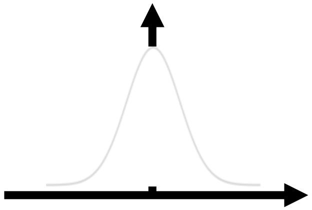


Spatial Parameter

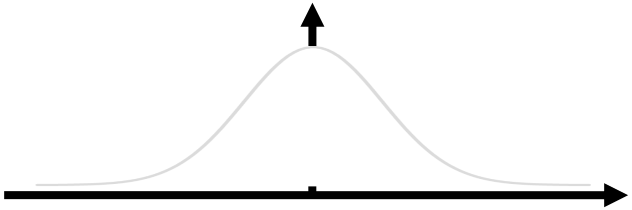
input

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\| \mathbf{p} - \mathbf{q} \|) I_{\mathbf{q}}$$

size of the window



small σ

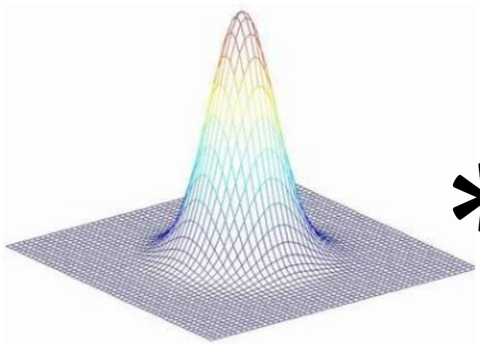


large σ

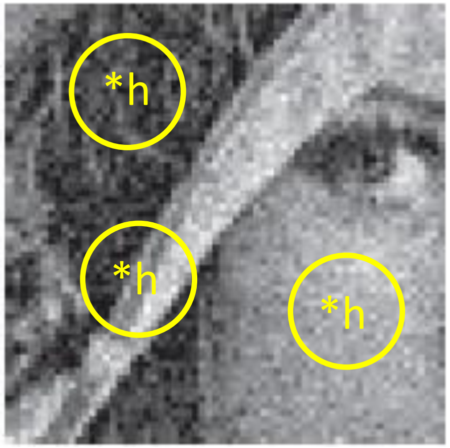
limited smoothing

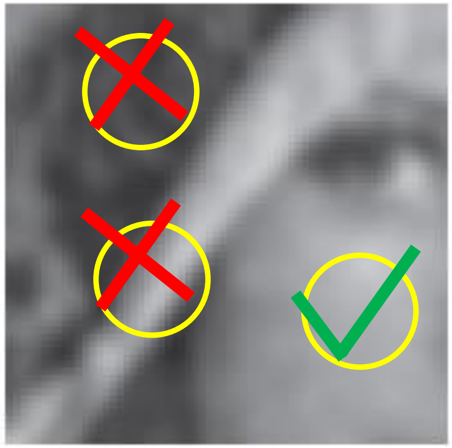
strong smoothing

Gaussian Smoothing



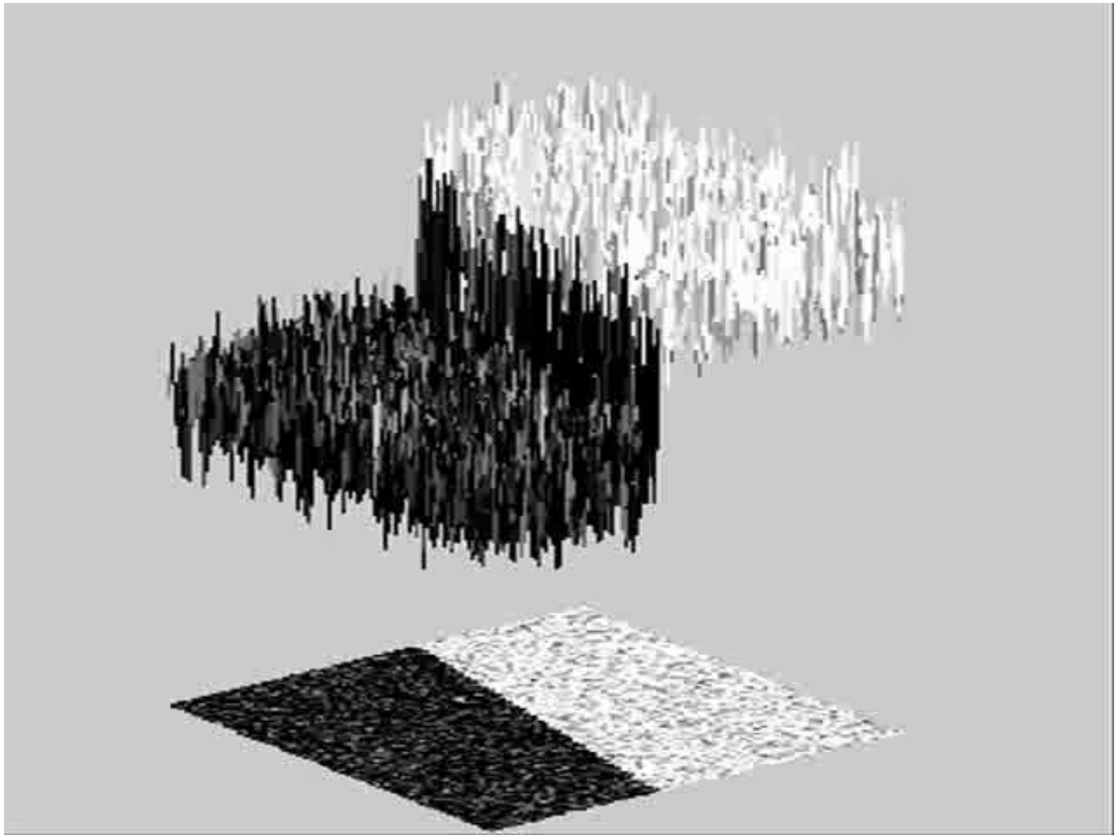
h





- $$\hat{x}(i) = \frac{1}{C_i}\sum_j y(j)e^{-\frac{\|i-j\|^2}{2\sigma^2}}$$

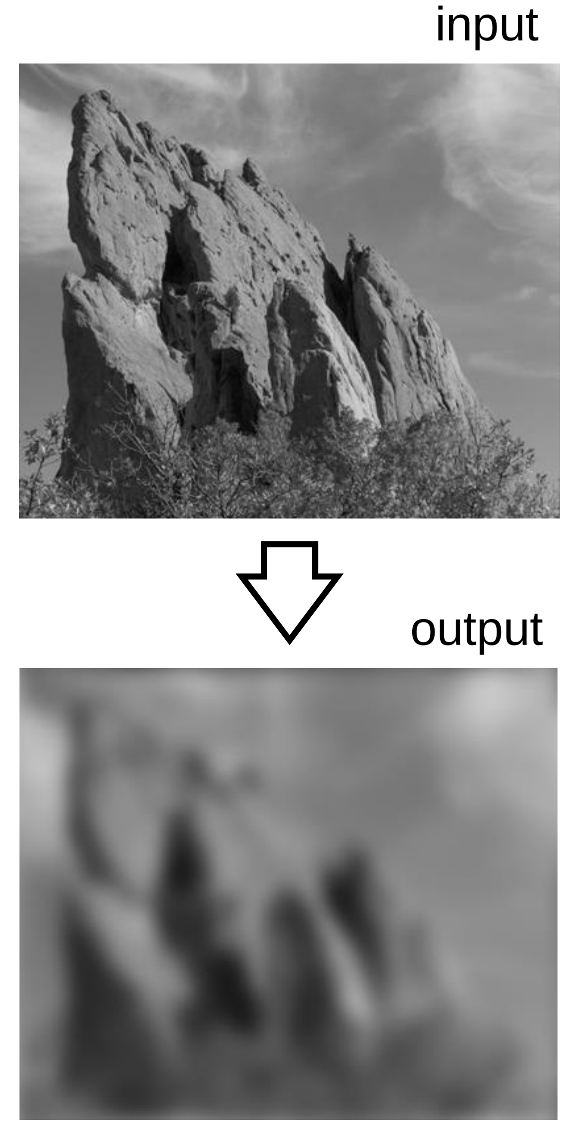
Toy Example



How can we preserve the fine details?

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much: **edges are blurred.**
  - Only spatial distance matters
  - No edge term



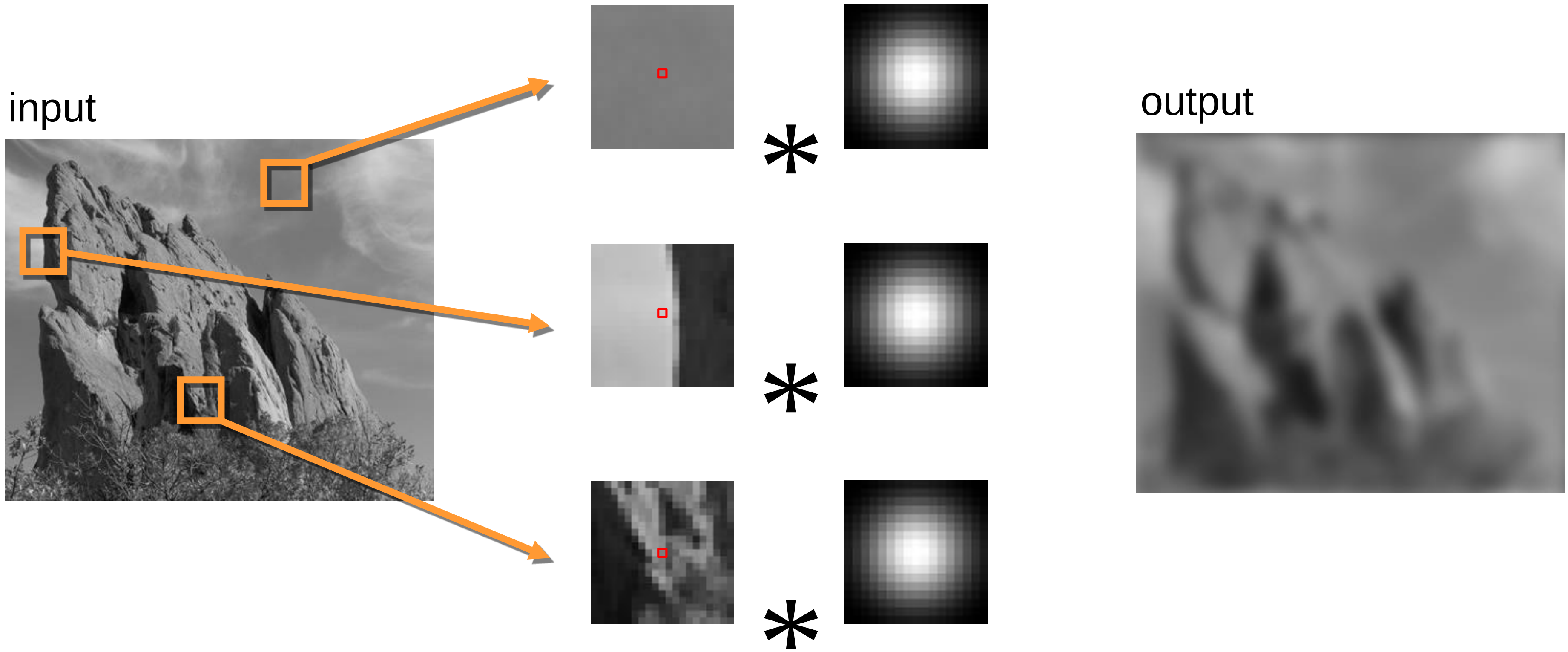
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} \underbrace{G_{\sigma}(\| \mathbf{p} - \mathbf{q} \|)}_{\text{space}} I_{\mathbf{q}}$$

Bilateral filtering

[Ben Weiss, Siggraph 2006]

Slides taken from Sylvain Paris, Siggraph 2007

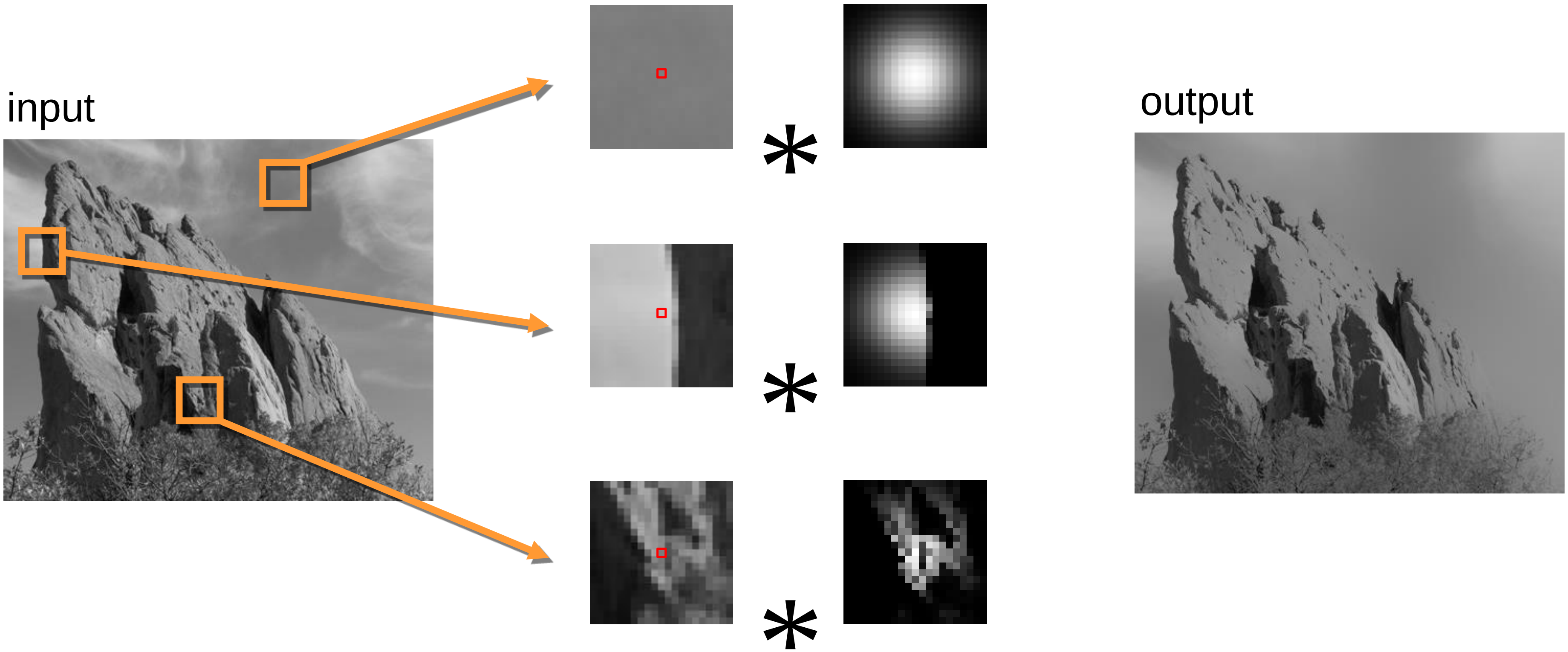
Gaussian Smoothing



Same Gaussian kernel everywhere
Averages across edges ⇒ blur

Slides taken from Sylvain Paris, Siggraph 2007

Bilateral Filtering



Kernel shape depends on image content
Avoids averaging across edges

Bilateral Filter Definition

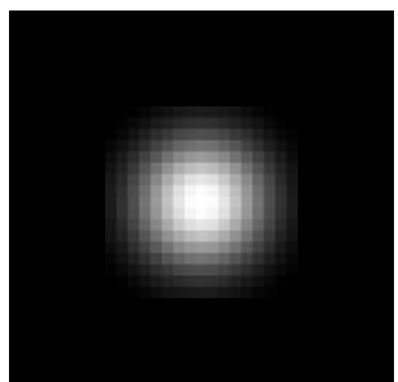
Same idea: **weighted average of pixels.**

new — not new — new

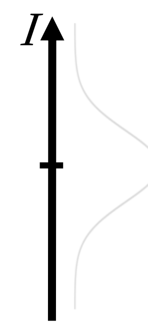
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \, I_{\mathbf{q}}$$

normalization factor

space weight

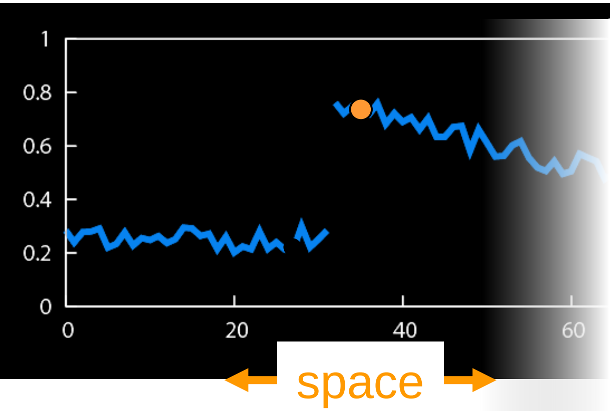


range weight



Gaussian Blur and Bilateral Filter

Gaussian blur

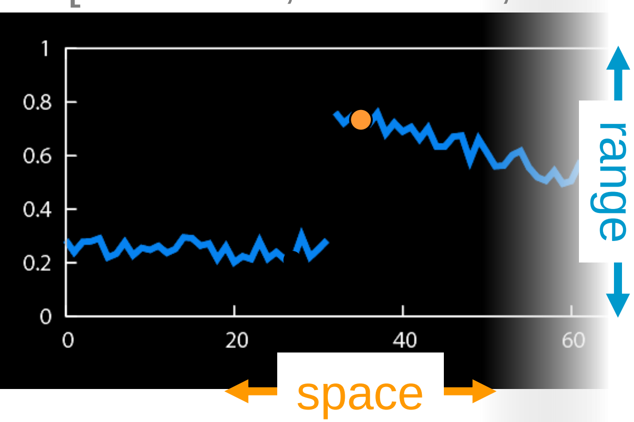


$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\| \mathbf{p} - \mathbf{q} \|) \, I_{\mathbf{q}}$$

spac
e

Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \, I_{\mathbf{q}}$$

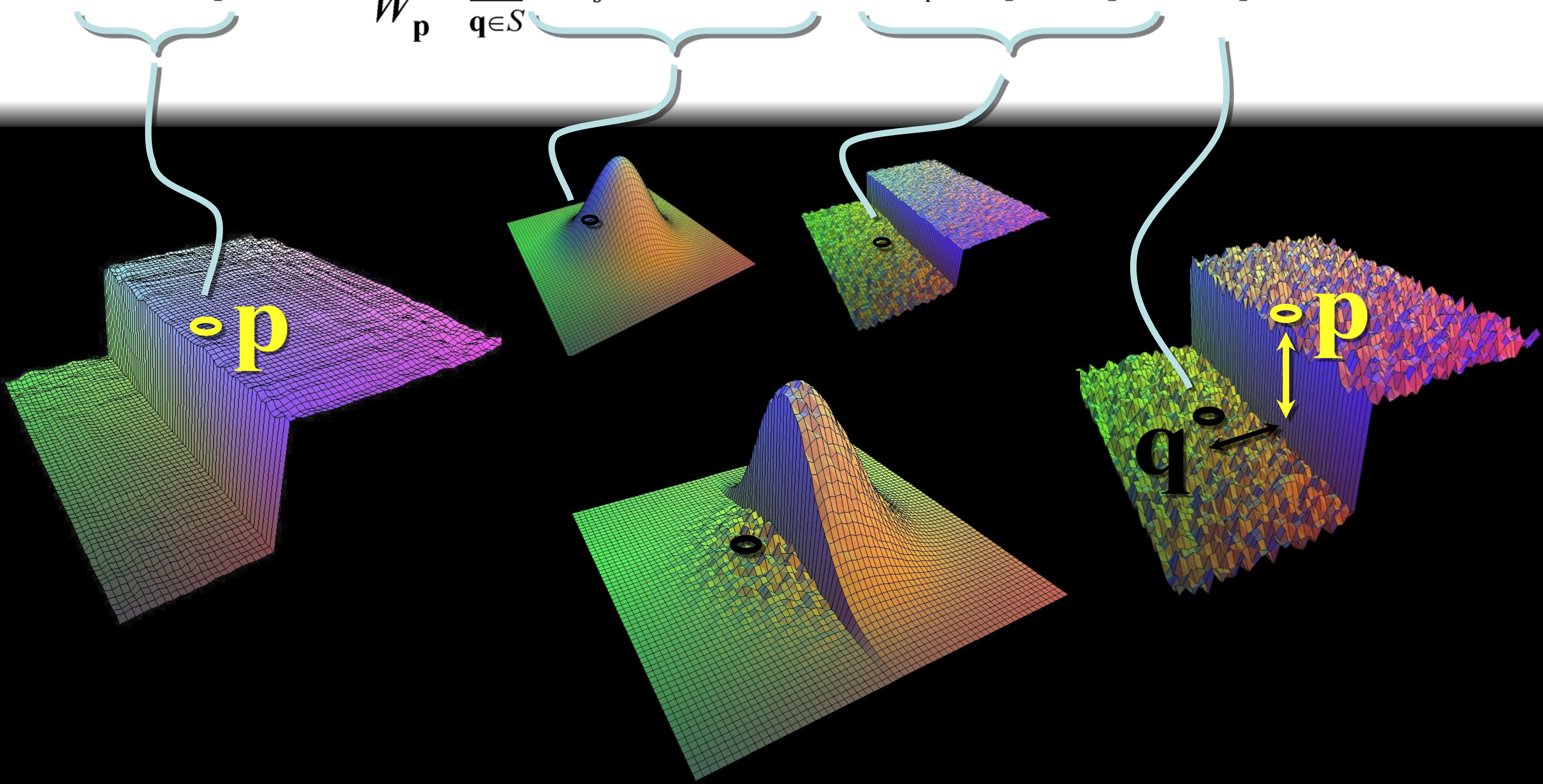
normalization

spac
e

range

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \; G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \; I_{\mathbf{q}}$$



reproduced from [Durand 02]

Space and Range Parameters

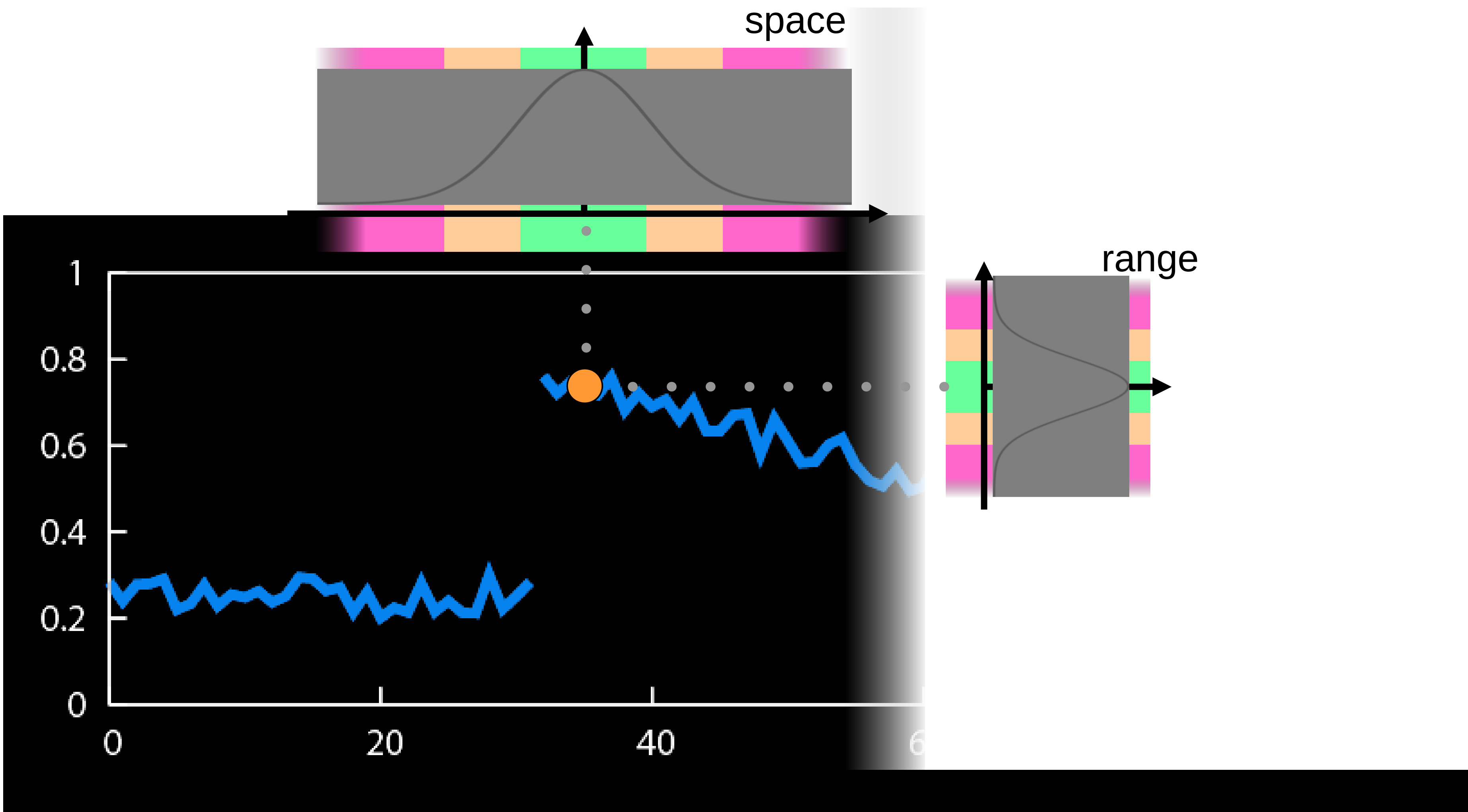
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \, I_{\mathbf{q}}$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.

- range σ_r : “minimum” amplitude of an edge

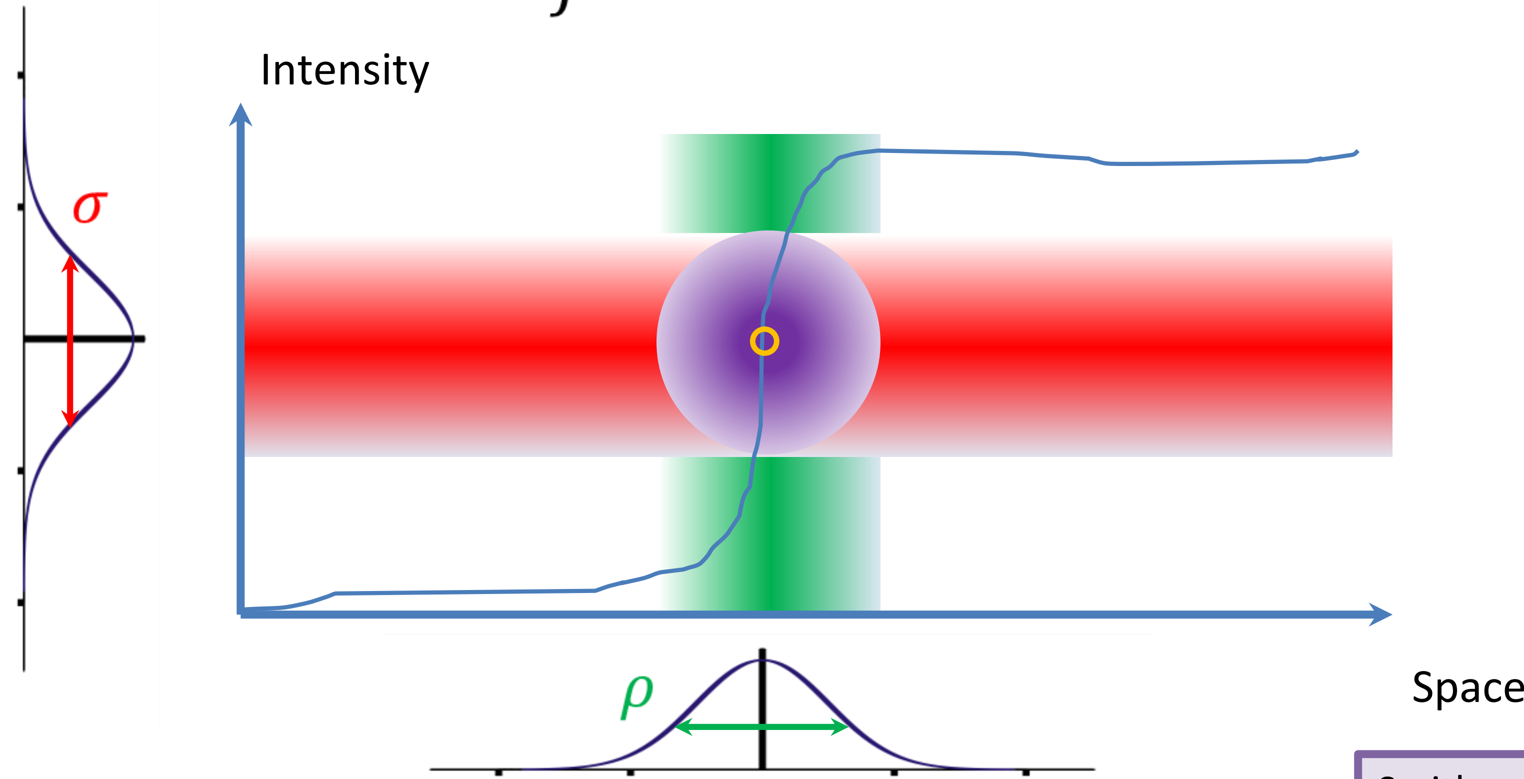
Influence of Pixels

Only pixels close in space and in range are considered.



Bilateral Filtering

- $$\hat{x}(i) = \frac{1}{C_i} \sum_j y(j) e^{-\frac{\|i-j\|^2}{2\rho^2}}$$



Smith and Brady (1997)

Exploring the Parameter Space

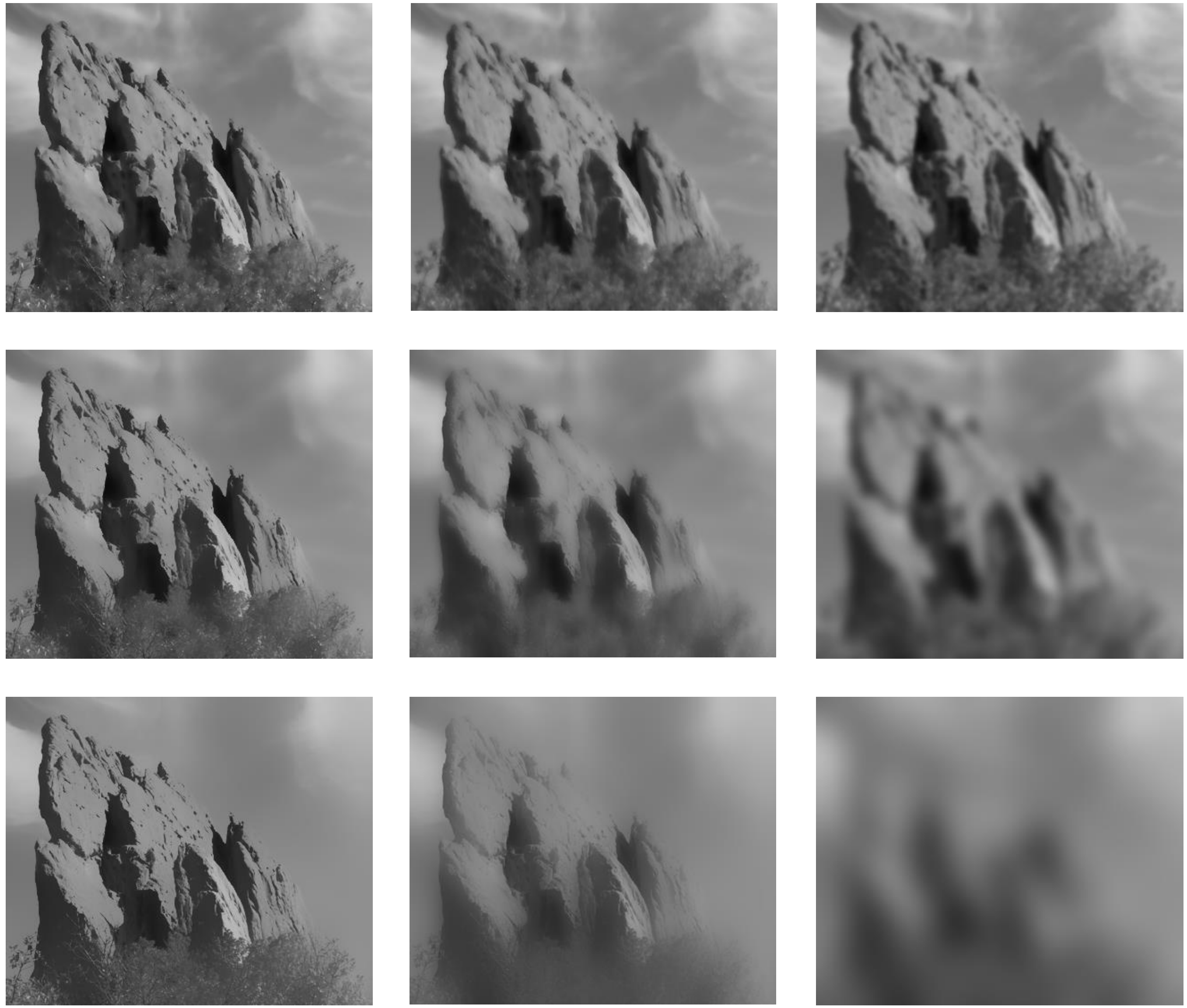
input

$\sigma_r = 0.1$ | $\sigma_r = 0.25$ | $\sigma_r = \infty$ (Gaussian blur)

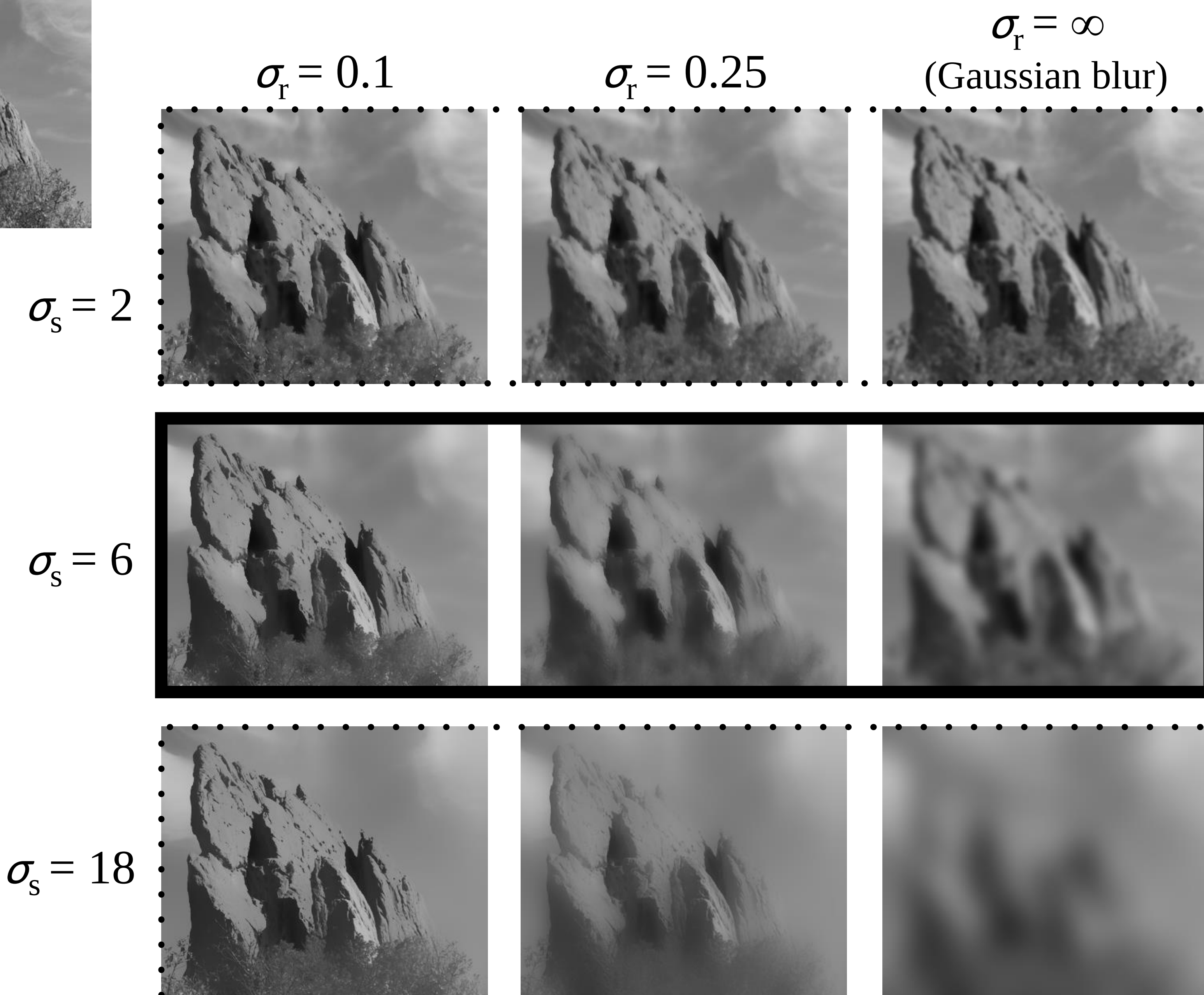
$\sigma_s = 2$

$\sigma_s = 6$

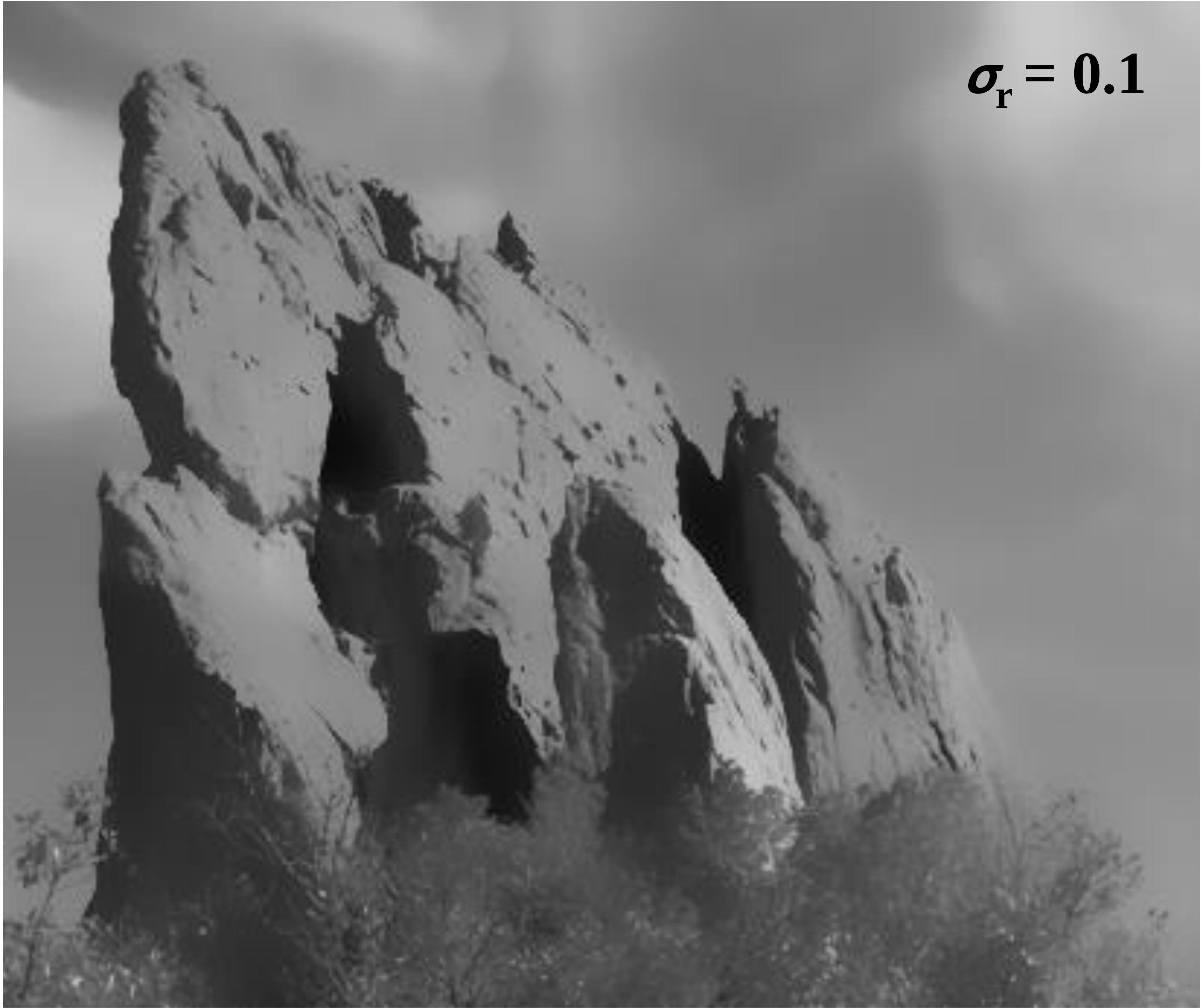
$\sigma_s = 18$

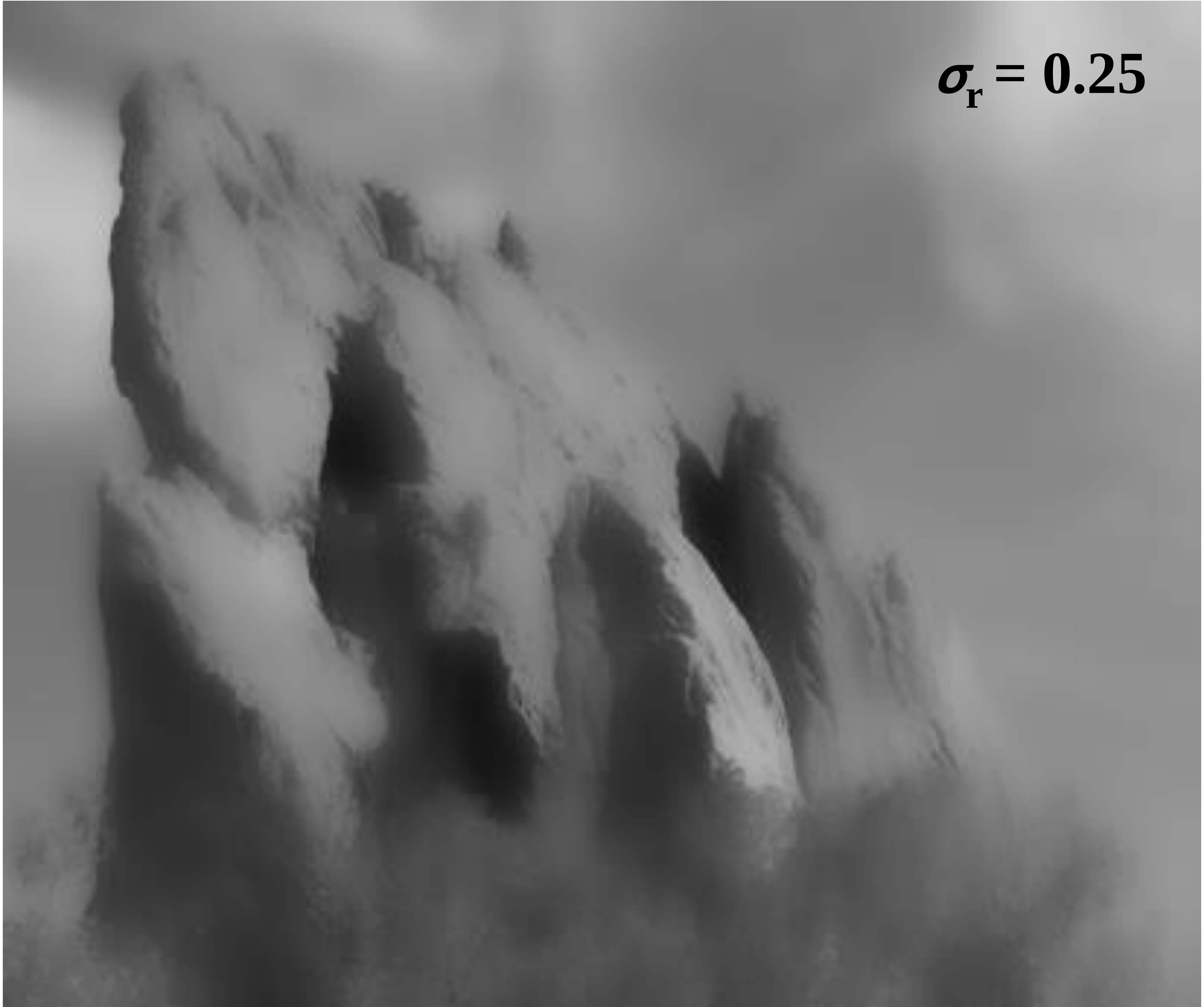


Varying the Range Parameter



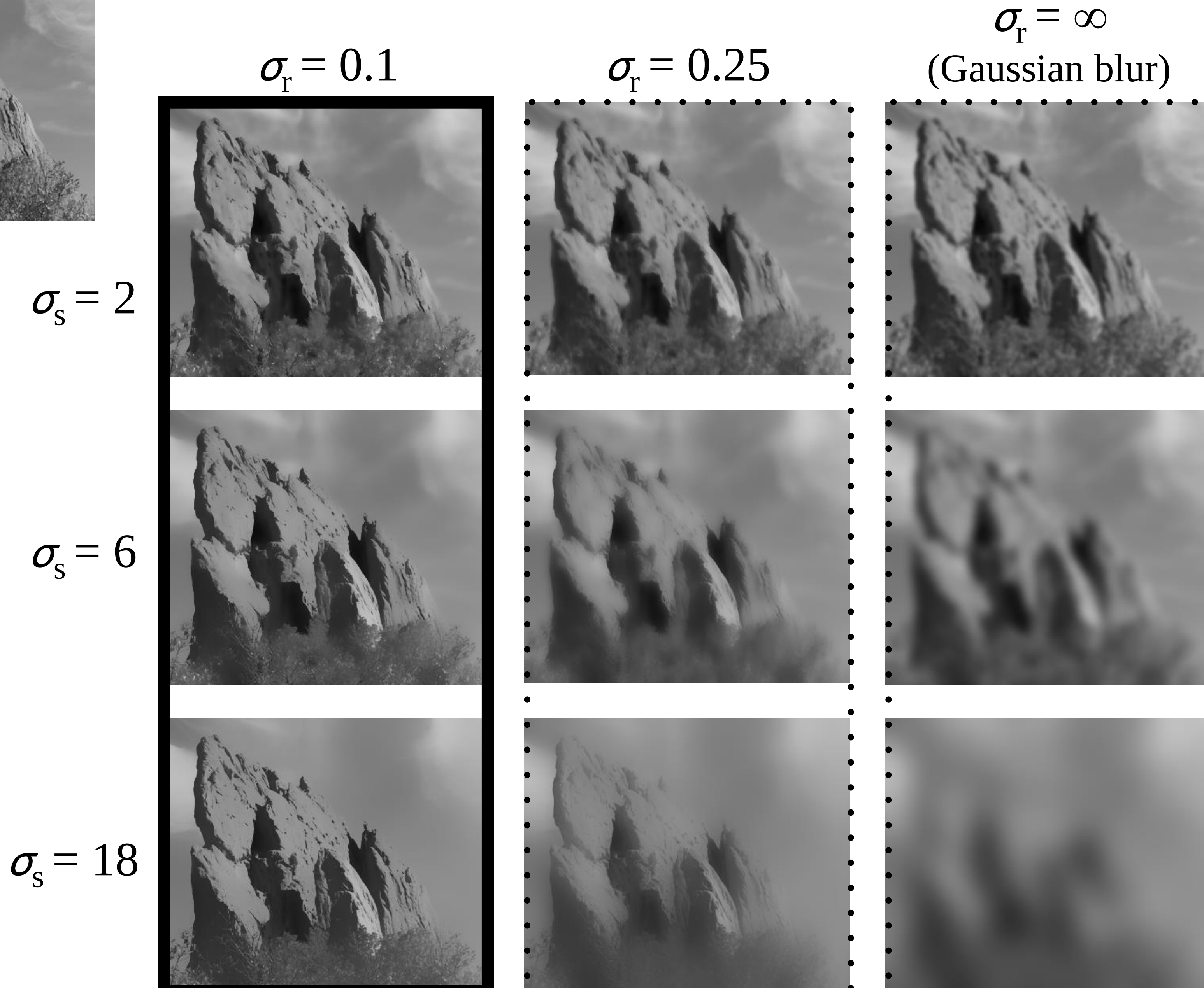

input


$\sigma_r = 0.1$


$\sigma_r = 0.25$


$\sigma_r = \infty$
(Gaussian blur)

Varying the Space Parameter



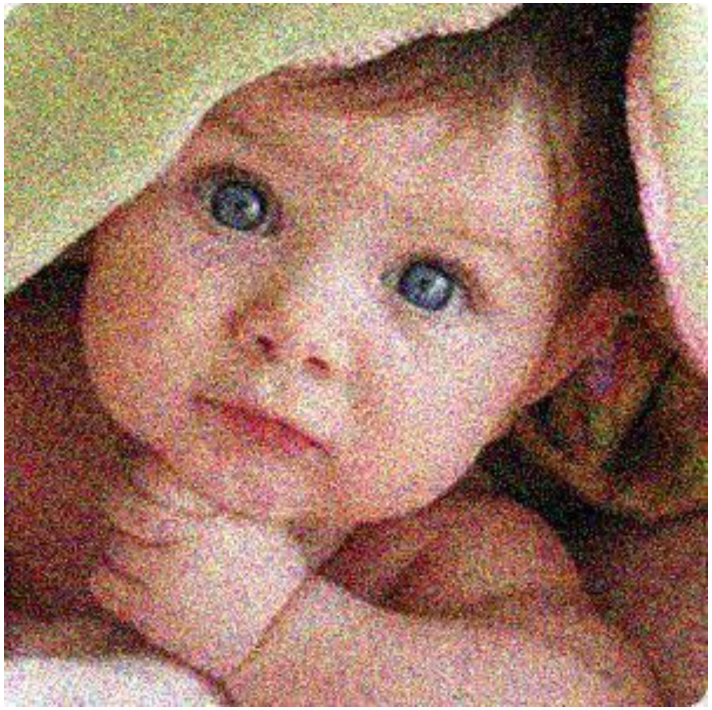

input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$

Which image do you prefer?



Reference

1. R. C. Gonzalez and R. E. Woods, *Digital Image Processing 2/E*. Upper Saddle River, NJ: Prentice-Hall, 2002, pp. 349-404.
2. S. Mallat, *Academic press - A Wavelet Tour of Signal Processing 2/E*. San Diego, Ca: Academic Press, 1999, pp. 2-121.
3. J. J. Ding and N. C. Shen, "Sectioned Convolution for Discrete Wavelet Transform," June, 2008.
4. Clecom Software Ltd., "Continuous Wavelet Transform," available in http://www.clecom.co.uk/science/autosignal/help/Continuous_Wavelet_Transfor.htm.
5. W. J. Phillips, "Time-Scale Analysis," available in http://www.engmath.dal.ca/courses/engm6610/notes/node4.html.


Any questions?