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RESEARCH ARTICLE

Proposed Capability Indices Based on Robust Estimation Compared with Classical Capability Indices

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ABSTRACT

A process capability study is a scientific and systematic procedure that uses control charts to detect and eliminate the unnatural causes of variation until a state of statistical control is reached. On the other hand, to meet the quality requirements of the final product, quality should be achieved at every stage of production. Another way of achieving good quality during production is to use statistical techniques at every stage of production. The purpose of this research is to apply it to process capacity indices in replace of the standard deviation estimator. The information, which is taken from the Coca-Cola/Erbil production process, illustrates the qualities of the beverage (750 mL). A Coca-Cola 'product's 100 observations are divided into 25 models. Employed both the standard deviation estimator-based and the robust Downton estimation-based process capability indices. It was determined that in this inquiry, the robust Downton estimation had better qualities than the standard division estimator because the robust Downton estimation process capacity index values were greater.

Keywords: Process capability index, robust estimation, quality control chart, standard deviation, specification

INTRODUCTION

Quality control is the application of techniques and actions to attain, maintain, and enhance product or service quality[1] and states that such charts are in the form of a graph that depicts the average output of data or a product when the process is statistically controlled.[2]

Process capability analysis has long been utilized as the gold standard performance criterion for assessing a process ability to meet customer requirements represented in certain specifications.[3]

Process capability is a strategic management tool that is utilized as a TQM tool and is essential to the management of an organization's operations. The process capability research helps with product design, acceptance standard selection, and operator and operations management process selection. Process capacity evaluation is a critical step in improving process quality. Such an effective tool supports product and process developers in making judgments on the creation of items or processes, assessing, and identifying competing suppliers, and finding the process that is the process quality bottleneck. Process capacity measures how effectively a process performs under unpredictable, daily circumstances. Its indications are intended to measure a process inherent variability and, in turn, indicate how well it operates. In a broad, non-theoretical sense, the presence of outliers might be taken as an indication that the process is out of statistical control because outliers are typically data from distributions distinct from the main set of data. In that regard, there are numerous mathematical techniques available to address the issue of outliers. These techniques are all based on solid statistics. Numerous applications of statistical process control analysis make use of robust statistics. Identifying control limits for control charts requires accurate estimations of process parameters, such as location, scale, etc.[4] The authors of Grznar *et al*.[5] and Mahmood[6] offer a methodology for outlier detection based on smoothing techniques. Different techniques for obtaining confidence intervals for polymerase chain reaction based on reliable estimations of non-normal data are discussed in Kocherlakota and Kocherlakota.[7] In the context of time series,

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Prasad and Bramorski[8] investigated the interplay between outliers and correlation structures. In addition, an index based on the notion of estimating non-conforming proportions was presented by Yeh and Bhattcharya.[9] In addition, they spoke about how to get bootstrap confidence intervals. When the observed random process deviates from normality, $C_{p,}$ and C_{pk}, two well-known process capability ratios, are examined for their resilience. When examining deviations from normalcy, large-scale simulation studies are validated using the distributions of predicted process capability ratios as a basis. The analytical findings and simulation studies serve as the foundation for suggested procedures. Because popular process capability indices lack robustness when deviations from normalcy are observed, it is advised to take the impact of process distributions into account before employing them. This paper intends to evaluate the robustness of the new capacity based on robust estimation ratios in the presence of outliers and a lack of normalcy. The data are collected from the manufacture of Coca-Cola in Erbil and pertain to the qualitative attributes of its 750 mL beverage items. A quality control chart (QCC) and a capability process are the two techniques, we employed in this paper**.**

QCCS

A process chart, also known as a QCC, is a graph that shows how, when a process is statistically controlled, the average value of the data (output) or product falls within the common or customary range of variation.

The first QCC was produced in 1924 by Shewhart[10] of Bell Telephone Laboratories, which he later amended with a colleague. In 1931, he released a comprehensive exposition of control charts.

Category of Control Charts

Control charts can be segregated into two categories, namely:

Variable control charts

When the goods being made are quantifiable, these charts are used to regulate the production process. For the purpose of creating charts, it is preferred that at least 25 samples and a minimum of 4 units per sample (apart from individual charts) should be used. Only one specific quality or trait may be applied to a variable chart.[11]

There are several variable control charts; however, the most significant ones include $\bar{x}$ -chart, SD-chart, R-chart, individual-chart, CUSUM-chart, MA-chart, MR-chart, GMA-chart, and RDC-chart.

Attributes control charts

A control chart for attributes is utilized when

a. Measurements are impossible (for instance, a flaw like dented cans).
b. Measurements are not feasible (such as time-consuming chemical analysis of raw materials).
c. A chart with many features (such as counts of various defect types) is used. In this situation, it is possible to combine all of the different qualities into one chart, or at most two or three charts, each of which would include the set of characteristics that best indicates their significance, namely minor, major, and crucial.

The attribute control charts are categorized as follows:

a. Defective or non-conforming chart. P-chart (fraction non-conforming)
b. np-chart (number non-conforming)
c. Defects or non-conformities charts. C-chart (number of non-conformities)
d. U-chart (average number of non-conformities).

Uses of control charts

Control charts can be used for the following purposes:

- It functions as a mechanism for early warning detection
- It is a tried-and-true method for boosting output
- It is successful in preventing defects
- It prevents needless process adjustments
- It offers diagnostic details
- It offers details about the processing power
- It aids in identifying the root causes of quality issues.

Robust Downton control chart

Downton was the first to offer a robust scale and an estimate for the standard deviation of a normal population, known as the Downton estimate. The Downton statistic is a credible estimator, according to the researchers.[12] Let X1, X2..., Xn represent a random sample of size n drawn from a normal distribution with mean and standard deviation; in other words, let XN(2) and the accompanying order statistic should be represented by X(1), X(2)..., X(n), where X(1)X(2)...... X(n). This article defines the estimator of Downton[13,14] as:

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[(i - \frac{1}{2}(n+1) \right] x_i \tag{1}$$

Where the unbiased estimator for σ is provided as $\hat{\sigma}$ =z3 $\bar{D}$, which is applied in this study.

$$\bar{D} = \frac{\sum_{i=1}^{m} D_i}{m} \tag{2}$$

(m): The subgroup's initial number and D are specified as in (1)

For the suggested dispersion chart, the commonly used 3-sigma control limits are therefore specified as:

$$\begin{aligned} UCL &= \bar{D} + 3z_3\bar{D} \\ T &= \bar{D} \\ LCL &= \bar{D} - 3z_3\bar{D} \end{aligned} \tag{3}$$

Where

$$z_3 = \frac{1}{\sqrt{n(n-1)}} \sqrt{n(\frac{1}{3}\Pi + 2\sqrt{3} - 4) + (6 - 4\sqrt{3} + \frac{1}{3}\Pi)} \tag{4}$$

Classical standard deviation chart

Since the sample is based on the process standard deviation, this chart regulates the process variability. The (S) 'chart's restricted and center lines are as follows:

$$UCL = \bar{S} + 3\sigma_S$$

$$T = \bar{S} = \frac{\sum_{j=1}^{m} S_i}{m} \tag{5}$$

$$LCL = \bar{S} - 3\sigma_S$$

Proposed process capability indices based on classic and robust estimation

The process capability index (PCI), which assesses a process capacity on the assumption that it is normally under statistical control and complies with specified criteria, is a measurement of a process capacity. To delve into the details, if X is a property of a process, check to see if it is inside the range (LSL, USL), whose ends are referred to as upper and lower specification limits. Given that X is a random variable, a PCI should be dependent on either the probability that X falls inside the specification interval or the average deviation of X from the specification constraints. In many cases, a technique will include these two requirements as equivalent portions. For example, consider the conventional process capability ratio.[15]

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \tag{6}$$

The C_{pk} index suggested by Chan *et al*.[16] are a measure of a process ability in comparison to the process average. It depends on the separation between the process average and the nearest specification limit, which is described as:

$$C_{pk} = \min\left(\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right) \tag{7}$$

Chan suggested an index termed C_{pm} that is labeled[17-19] as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\hat{\sigma}^2 + (\bar{x} - T)^2}} \tag{8}$$

In (1992) Kotz and Johnson posited an index, named, C_{pmk} that is labeled as:

$$C_{pmk} = \frac{\min(USL - \bar{x}, \bar{x} - LSL)}{3\sqrt{\hat{\sigma}^2 + (\bar{x} - T)^2}} \tag{9}$$

The C_p index dependent on the robust Downton estimator is defined by: [20-23]

$$C_p = \frac{USL - LSL}{6(\bar{D})} \tag{10}$$

The C_{pk} index founded on robust Downton estimator is defined by

$$C_{pk} = \min\left(\frac{USL - \bar{\bar{x}}}{3\bar{D}}, \frac{\bar{\bar{x}} - LSL}{3\bar{D}}\right) \tag{11}$$

The C_{pm} index established on robust Downton estimator is defined by

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\bar{D})^2 + (\bar{\bar{x}} - T)^2}} \tag{12}$$

The C_{pmk} index built on robust Downton estimator is defined by

$$C_{pmk} = \frac{\min(USL - \bar{\bar{x}}, \bar{\bar{x}} - LSL)}{3\sqrt{(\bar{D})^2 + (\bar{\bar{x}} - T)^2}} \tag{13}$$

RESULTS

The following subsections present the results corresponding to the application of the proposed capability indices (and their quality assessment methodology) to the real data.

Real Data

The data are gathered from Coca-Cola/Erbil production and represent its drink's quality characteristic (750 mL) products. The (100) observations of a 750 mL Coca-Cola product are classified into 25 models, with each having four observations, as indicated in Table 1.

Phase 1: Scale quality control charts to monitor process dispersion

The points in Figure 1 do not all fall within the range of control. For the same qualities of quality from which we received the data, the above chart and D-chart's sensitivity to spot problems in the process, therefore, may be trusted and utilized going forward for the purposes of controlling and monitoring future production.

[Figure 2] demonstrates that every point is contained within the range of control. This indicates that the aforementioned chart may be trusted and used going forward for similar qualities while collecting the data aiming at controlling and monitoring upcoming output.

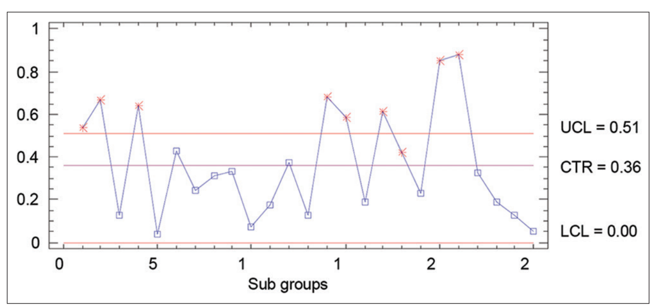

Figure 1: Robust D-control chart (RDCC)

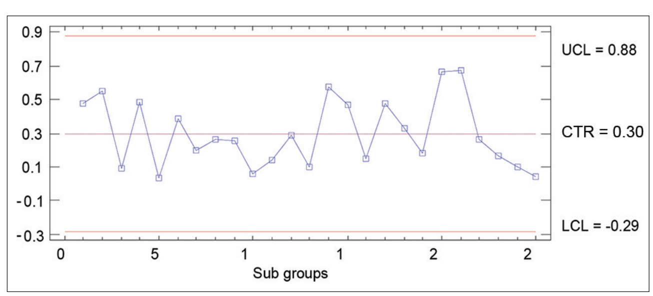

Figure 2: Classical S-control chart (CSDC)

Table 1: Coca-Cola drink product

Subgroups	p1	p2	p3	p4	S.D	D
1	750.14	750.36	750.36	751.36	0.546596	0.540598
2	750.78	750.86	751.86	751.96	0.63148	0.669987
3	751.12	751.22	751.28	751.38	0.108934	0.124036
4	750.02	750.36	750.88	751.28	0.556747	0.637822
5	750.48	750.48	750.5	750.56	0.037859	0.038391
6	750.08	750.2	750.2	751.04	0.443621	0.425389
7	750.4	750.42	750.7	750.86	0.223532	0.245024
8	749.56	750.02	750.2	750.2	0.302159	0.310073
9	750.34	750.49	750.7	751.02	0.294661	0.332211
10	750.5	750.54	750.54	750.66	0.069282	0.070898
11	750.52	750.62	750.66	750.9	0.161142	0.174268
12	750.16	750.42	750.74	750.9	0.330404	0.37498
13	750.34	750.48	750.54	750.6	0.111355	0.124036
14	750.32	750.44	750.68	751.78	0.667008	0.682253
15	750.6	750.62	751.28	751.7	0.536284	0.58452
16	750.26	750.3	750.52	750.62	0.173109	0.191886
17	750.24	750.84	750.96	751.58	0.549272	0.611425
18	750.16	750.52	750.92	750.98	0.382405	0.422198
19	750.54	750.62	750.8	751	0.204613	0.230313
20	750.69	751.4	752.18	752.36	0.76787	0.854748
21	750.32	750.9	751.32	752.16	0.774145	0.877116
22	750.8	750.8	751.08	751.44	0.303535	0.324784
23	750.12	750.28	750.22	750.56	0.188591	0.186143
24	750.26	750.4	750.44	750.54	0.116046	0.129956
25	750.44	750.5	750.5	750.56	0.04899	0.053174

Table 2: Capability indices comparison based on classical with capability indices based on the robust estimation

Robust process capability ratios	Classical process capability ratios
C_p = 1.8	C_p = 1.3
C_{pk} = 1.09	C_{pk} = 1.03
C_{pm} = 1.17	C_{pm} = 1.07
C_{pmk} = 0.89	C_{pmk} = 0.83

It is shown in Table 2 that process capability indices are derived from both classical and robust estimation.

CONCLUSION

1. The robust Downton estimation outperformed the other standard deviation when we compared scale estimations. Consider that the robust Downton estimator has higher process capability index values than the estimator based on that, indicating that the Downton estimator has superior features. The suggested Downton estimate should be used instead of the standard deviation.
2. According to comparative evaluations, the new ratios are reliable measurements for estimating the genuine degree of process capability under typical conditions.
3. The control chart, depending on the Downton estimator, has a narrower space between the upper and lower bounds than the sigma estimation control chart. So much so that the Downton estimator can be considered a major approximation for lowering the amount of inaccuracy.

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