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Comparison between two Estimators by Using Process Capability with Application

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ABSTRACT

Process capability is the long-term performance level of the process after it has been brought under statistical control. In other words, process capability is the range over which the natural variation of the process occurs as determined by the system of common causes.
Robust estimator is an estimator which is insensitive to changes in the underlying distribution and also resistant against the presence of outliers.

In this paper proposes the using Downton estimator in the process capability indices and compare with $(\hat{\sigma})$ estimator in the Process Capability Indices to justify the efficiency. Concluded that in this study, Downton estimator have better property than the other $(\hat{\sigma})$ estimator because that the process Capability Indices values based on Downton estimator greater than the process Capability Indices values based on $(\hat{\sigma})$ estimator.We also found the process Capability Indices based on Downton estimator can be used as an alternative instead of the process Capability Indices values based on $(\hat{\sigma})$ estimator.

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INTRODUCTION

Process capability measurements allow us to summarize process c

apability in terms of meaningful percentages and metrics to predict the extent to which the process will be able to hold tolerance or customer requirements.

You can compute how often the process will meet the specification or the expectation of your customer, you may learn that bringing your process under statistical control requires fundamental changes - even redesigning and implementing a new process that eliminates the sources of variability now at work.

It helps you choose from among competing processes, the most appropriate one for meeting customers' expectation knowing the capability of your processes. You can specify better the quality performance requirements.

The classical process capability ratio (**Kane, 1986**):

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \quad \ldots. (1)$$

Where

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

c_4 :

Factors for Computing Central Lines and Control Limits based on sample size (n) .

The C_{pk} index proposed by (**Sullivan, 1985**) is a measure of the capability of a process in relation to the process average. It is based on the distance between the process average and the closest specification limit, and is defined as:

$$C_{pk} = \min\left(\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right) \ldots. (2)$$

$$C_{pk_U} = \frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}$$

$$C_{pk_L} = \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}$$

(**Chan, 1988**) proposed another index, called C_{pm} which is defined as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\hat{\sigma}^2 + (\bar{x} - T)^2}} \qquad \ldots.(3)$$

(**Kotz, S. & Johnson.; 1992**) proposed another index, called, C_{pmk} which is defined as:

$$C_{pmk} = \frac{\min(USL - \bar{x}, \bar{x} - LSL)}{3\sqrt{\hat{\sigma}^2 + (\bar{x} - T)^2}} \qquad \ldots.(4)$$

Robust Scale Downton Estimator.

The Robust Scale Downton estimator was first introduced by Downton as an estimator for the standard deviation of a normal population. (**Barnett, Mullen & Saw**) showed that Downton statistic is an unbiased estimator of (σ). Let X_1, X_2 ,……, X_n represent a random sample of size n from a normal distribution with mean μ and standard deviation (σ); that is, let $X \sim N(\mu,\sigma 2)$ and the corresponding order statistic be denoted by $X_{(1)},X_{(2)},\ldots\ldots X_{(n)}$ where $X_{(1)} \le X_{(2)} \ldots\ldots.. \le X_{(n)}$. The Downton estimator is defined (**Downton, F. 1966**) (**Abbasi, S.A. & Miller, A. 2013**) as:

$$\left.\begin{aligned} D &= \sqrt{\pi}\sum_{i=1}^{n}\frac{(2i-n-1)x_i}{n(n-1)} \\ D &= \frac{2\sqrt{\pi}}{n(n-1)}\sum_{i=1}^{n}\left[(i-\frac{1}{2}(n+1)\right]x_i \end{aligned}\right\} \ldots (5)$$

Where the unbiased estimator for (σ) is given as $\hat{\sigma} = \overline{D}$, which is used in this study.

$$\overline{D} = \frac{\sum_{i=1}^{m} D_i}{m}$$

Where

m: is a preliminary number of the subgroups.

D: is defined as in (5).

Process Capability Indices Based on Robust Scale Downton Estimator

Let x_{ij} represent a random sample of size n taken over m subgroup, i = 1, 2…, n and j = 1, 2,…, m. The sample are assumed to be independent and taken from a continuous identical distribution functions. If (σ^2) is unknown, then an unbiased estimate of (σ^2) is the sample variance (s^2). In practice, the normality assumption is often violated by real life data, therefore, using (s^2) as an estimate of (σ^2) will affect the process capability indices and thus this might leads to wrong signal and invalid inference. The C_p index based on Downton estimator will be defined by (**Adeoti, O. A, Olaomi, J.O. & Adekeye, K.S. 2016**):

$$C_p = \frac{USL - LSL}{6(\overline{D})} \quad \text{…. (6)}$$

The C_{pk} index based on Downton estimator will be defined by

$$C_{pk} = \min\left(\frac{USL - \bar{\bar{x}}}{3\overline{D}}, \frac{\bar{\bar{x}} - LSL}{3\overline{D}}\right) \quad \text{…. (7)}$$

The C_{pm} index based on Downton estimator will be defined by

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\overline{D})^2 + (\bar{\bar{x}} - T)^2}} \quad \text{…. (8)}$$

The C_{pmk} index based on Downton estimator will be defined by

$$C_{pmk} = \frac{\min(USL - \bar{\bar{x}}, \bar{\bar{x}} - LSL)}{3\sqrt{(\overline{D}\,)^2 + (\bar{\bar{x}} - T)^2}} \quad \text{…. (9)}$$

Where $T = \frac{USL + LSL}{2}$

APPLICATION

We collected the data from the factory (Coca-Cola /Erbil) and the data representing the quality properties of drink (750 ml) for Coca-Cola product. We used (100) observations drink (750 ml) for Coca-Cola product, and divided into (25) samples and each sample consisting of (4) observations as shown in table (1):

Table1: drink (750 ml) for (Coca-Cola /Erbil) product

sub groups	x1	x2	x3	x4	x-bar	S	D
1	750.14	750.36	750.36	751.36	750.555	0.546596	0.540598
2	750.78	750.86	751.86	751.96	751.365	0.63148	0.669987
3	751.12	751.22	751.28	751.38	751.25	0.108934	0.124036
4	750.02	750.36	750.88	751.28	750.635	0.556747	0.637822
5	750.48	750.48	750.5	750.56	750.505	0.037859	0.038391
6	750.08	750.2	750.2	751.04	750.38	0.443621	0.425389
7	750.4	750.42	750.7	750.86	750.595	0.223532	0.245024
8	749.56	750.02	750.2	750.2	749.995	0.302159	0.310073
9	750.34	750.49	750.7	751.02	750.6375	0.294661	0.332211
10	750.5	750.54	750.54	750.66	750.56	0.069282	0.070898
11	750.52	750.62	750.66	750.9	750.675	0.161142	0.174268
12	750.16	750.42	750.74	750.9	750.555	0.330404	0.37498
13	750.34	750.48	750.54	750.6	750.49	0.111355	0.124036
14	750.32	750.44	750.68	751.78	750.805	0.667008	0.682253
15	750.6	750.62	751.28	751.7	751.05	0.536284	0.58452
16	750.26	750.3	750.52	750.62	750.425	0.173109	0.191886
17	750.24	750.84	750.96	751.58	750.905	0.549272	0.611425
18	750.16	750.52	750.92	750.98	750.645	0.382405	0.422198
19	750.54	750.62	750.8	751	750.74	0.204613	0.230313
20	750.69	751.4	752.18	752.36	751.6575	0.76787	0.854748
21	750.32	750.9	751.32	752.16	751.175	0.774145	0.877116
22	750.8	750.8	751.08	751.44	751.03	0.303535	0.324784
23	750.12	750.28	750.22	750.56	750.295	0.188591	0.186143
24	750.26	750.4	750.44	750.54	750.41	0.116046	0.129956
25	750.44	750.5	750.5	750.56	750.5	0.04899	0.053174

$$\overline{D} = \frac{\sum_{i=1}^{m} D_i}{m} = 0.36$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{0.3411}{0.9213} = 0.3702$$

$$\therefore \overline{D} < \hat{\sigma}$$

Construction Control Charts for data

Both charts are used for controlling the mean level in the data of (drink (750 ml)). The horizontal axis of the chart represents samples sequence, while the vertical axis represents the quality characteristic.

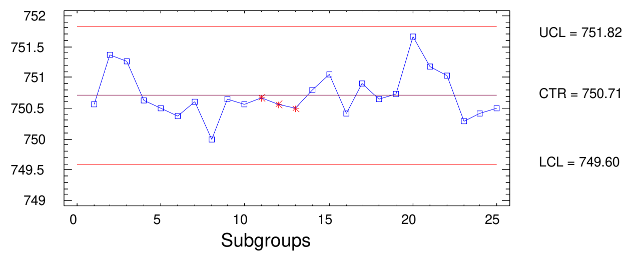

Figure (1): X –bar chart based on $(\hat{\sigma})$

Figure (1) shows that all the points are fallen within the limits of control. This means that the above chart can be relied upon and used in the future for the same properties of quality from which we obtained the data for the purpose of control and monitoring of future production.

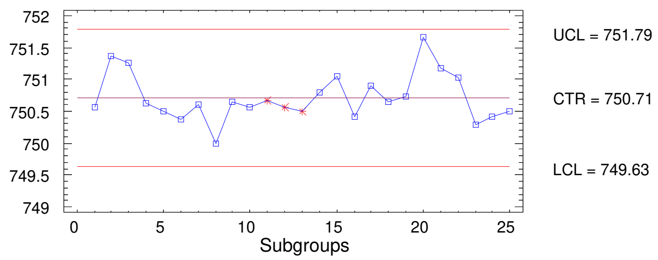

Figure (2): X –Bar chart based on Downton estimator

Figure (2) shows that all the points are fallen within the limits of control. This means that the above chart can be relied upon and used in the future for the same properties of quality from which we obtained the data for the purpose of control and monitoring of future production.

Table2. Comparison between Capability Indices Based on Downton estimator and Capability Indices Based on $(\hat{\sigma})$

Downton estimator	$(\hat{\sigma})$ estimator
$C_p = \frac{USL-LSL}{6(\bar{D})} = \frac{752.36-749.56}{6(0.36)} = 1.3 > 1$	$C_p = \frac{USL-LSL}{6(\hat{\sigma})} = \frac{752.36-749.56}{6(0.3702)} = 1.2 > 1$
$C_{pk} = \min\left(\frac{USL-\bar{\bar{x}}}{3(\bar{D})}, \frac{\bar{\bar{x}}-LSL}{3(\bar{D})}\right)$ $C_{pk} = \min\left(\frac{752.36-750.71}{3(0.36)}, \frac{750.71-749.56}{3(0.36)}\right)$ $C_{pk} = \min(1.53, 1.06)$ $C_{pk} = 1.06 > 1$	$C_{pk} = \min\left(\frac{USL-\bar{\bar{x}}}{3(\hat{\sigma})}, \frac{\bar{\bar{x}}-LSL}{3(\hat{\sigma})}\right)$ $C_{pk} = \min\left(\frac{752.36-750.71}{3(0.3702)}, \frac{750.71-749.56}{3(0.3702)}\right)$ $C_{pk} = \min(1.48, 1.03)$ $C_{pk} = 1.03 > 1$
$C_{pm} = \frac{USL-LSL}{6\sqrt{(\bar{D})^2+(\bar{\bar{x}}-T)^2}}$ $C_{pm} = \frac{752.36-749.56}{6\sqrt{(0.36)^2+(750.71-750.96)^2}}$ $C_{pm} = 1.07 > 1$	$C_{pm} = \frac{USL-LSL}{6\sqrt{\hat{\sigma}+(\bar{\bar{x}}-T)^2}}$ $C_{pm} = \frac{752.36-749.56}{6\sqrt{(0.3702)^2+(750.71-750.96)^2}}$ $C_{pm} = 1.04 > 1$
$C_{pmk} = \frac{\min(USL-\bar{\bar{x}}, \bar{\bar{x}}-LSL)}{3\sqrt{(\bar{D})^2+(\bar{\bar{x}}-T)^2}}$ $C_{pmk} = \frac{\min\left(1.65, \quad 1.15\right)}{3\sqrt{(0.36\ )^2+(750.71-750.96)^2}}$ $C_{pmk} = 0.88 < 1$	$C_{pmk} = \frac{\min(USL-\bar{\bar{x}}, \bar{\bar{x}}-LSL)}{3\sqrt{(\hat{\sigma})^2+(\bar{\bar{x}}-T)^2}}$ $C_{pmk} = \frac{\min\left(1.53, \quad 1.15\right)}{3\sqrt{(0.36\ )^2+(750.71-750.96)^2}}$ $C_{pmk} = 0.85 < 1$

II.CONCLUSION

1- When we compared scale estimate, Downton estimator have better property than the other $(\hat{\sigma})$ estimator. Reflect that the process Capability Indices values based on Downton estimator greater than the process Capability Indices values based on $(\hat{\sigma})$ estimatorthat is mean Downton estimator have better property than the other $(\hat{\sigma})$ estimator.It is recommended to use proposed Downton estimator as an alternative to $(\hat{\sigma})$ estimator.

2- The distance between the upper and lower limits in the control chart based on Downton estimator is less than the control chart based on $(\hat{\sigma})$ estimator. This means that the Downton estimator is an important estimate of reducing the size of the error go so far as to.

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