Seminar entitled by:

Fuzzy system and its generalizations with some applications: Human-like decision making

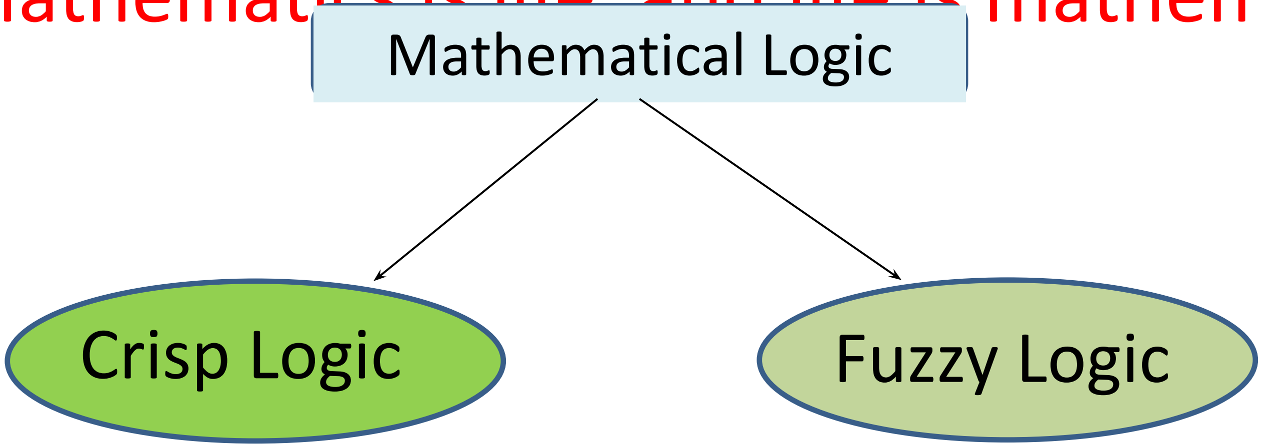
. By Dr. Salem Saleh, April 2022

Topics of this Seminar

- Definition of Fuzzy sets, Operations of fuzzy set, Fuzzy number and operations, Extension principle
- Fuzzy rules and Fuzzy control.
- Some generalizations of fuzzy sets such as:
- Soft sets and fuzzy soft sets.
- Some applications of fuzzy, soft, and fuzzy soft sets in: Computer Science and Engineering (decision making).

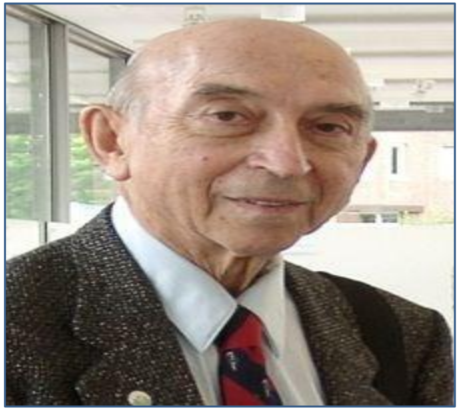
History of fuzzy logic

Its known that the mathematical Logic is the base of all topics in Mathematics which is the life, according to the famous Pythagorean saying, "Mathematics is life, and life is mathematics."



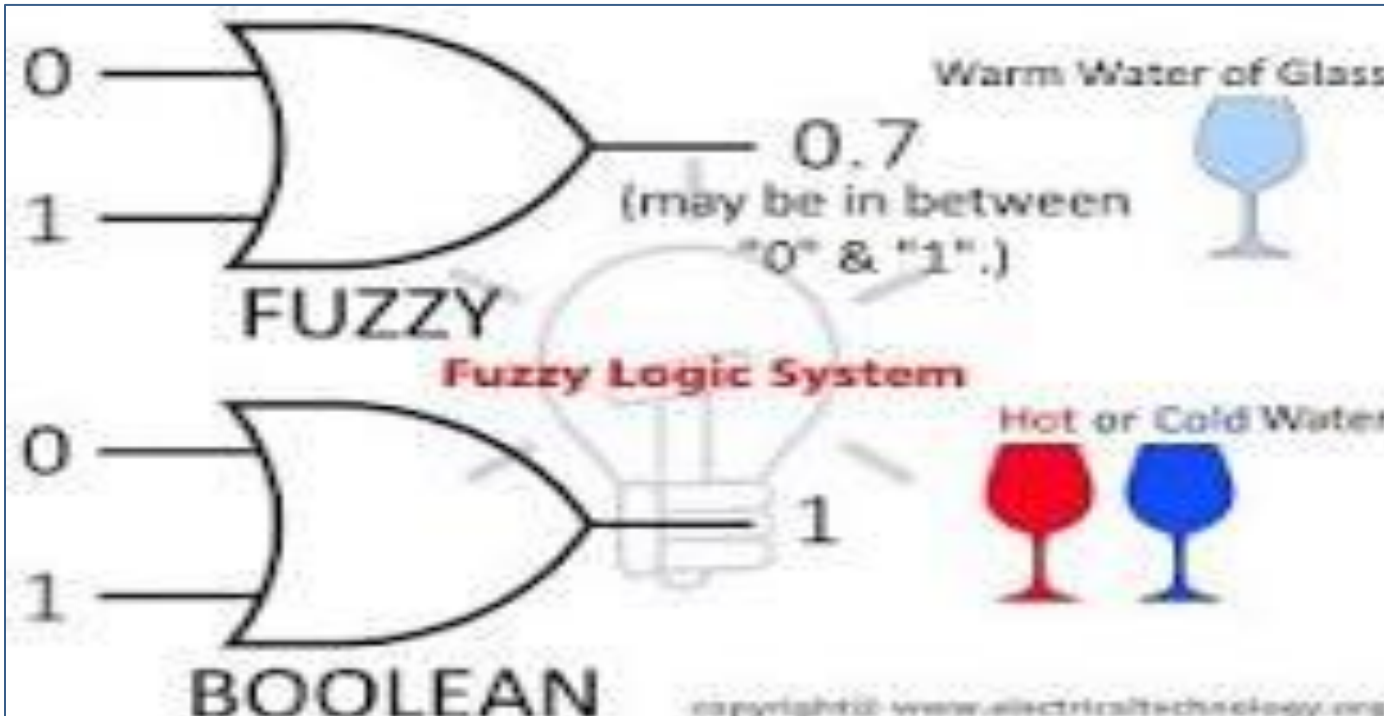
1- History of fuzzy logic

- The opposite word of fuzzy is crisp. **Fuzzy** means: un-clear or uncertain, **crisp** means: clear, clean, and sharp.
- Fuzzy logic was given by Zadeh in 1965, and was applied in steam engine control by Mamdani in 1974
- In crisp logic any statement take only one value either 1 or 0 that is, the statement can be {true or false}
- The fuzzy logic is a form of many-valued logic, where the truth value of any statement can be take 0 or 1 or any real



Prof. Lotfy Zadah

For the difference between crisp and fuzzy logic,
Let us give the following example



2- Crisp set vs. Fuzzy set

.Let X be an universe set

Crisp Set.
Is a set characterized by the function $\chi_A: X \longrightarrow \{0,1\}$.
Where χ_A is called the characteristic function of A
The range of χ_A is {T, F}

Fuzzy Set.
A fuzzy set A is a function $A: X \longrightarrow [0,1]$ and can given by $A = \{(x, A(x)): x \ in \ X\}$.
Where $A(x)$ is called the membership degree of x in A

Examples of fuzzy set

- ✔ The tall of people, old people, kind people
- ✔ temperature is hot, just good, a little bit cold

Example of Fuzzy set

- Let X be the set of stuff at CSD.

X= {Shukur, Lith, Khaled, Reem, Abdullah, Soma, Alya'a, Hasan, Salem}

The fuzzy set on X, A="Tall persons" we can write it as:

A={(0.7,Shukur),0.6),Khaled),(0.9, Ali),(0.5,Lith),(0.54,Reem),

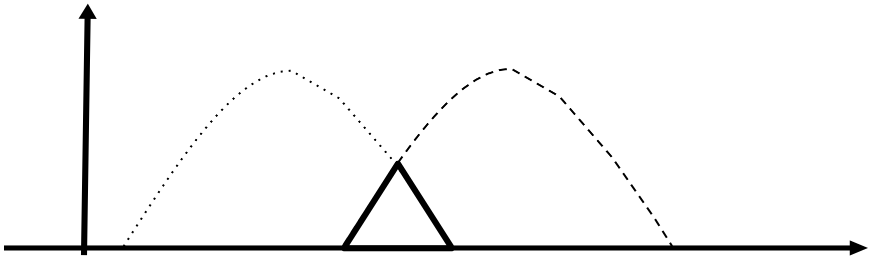
- So that, we can input the data of fuzzy set for "tall person" concept in computer by the following matrix:

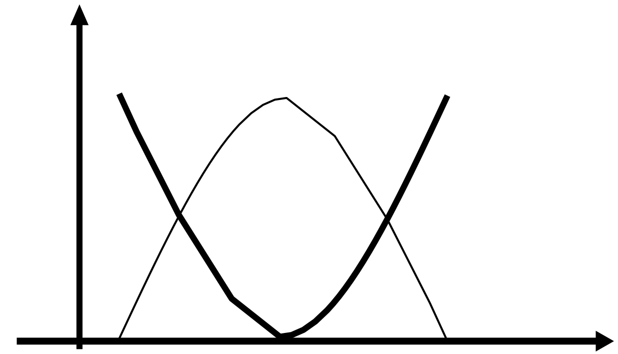
x(stuff)	$A(x)$ memb.Degree of tall
Shukur	0.8
Lith	0.5
Khaled	0.6
Abdullah	0.75
Alya'a	0.8
Reem	0.54
Ali	0.8
Salem	0.45

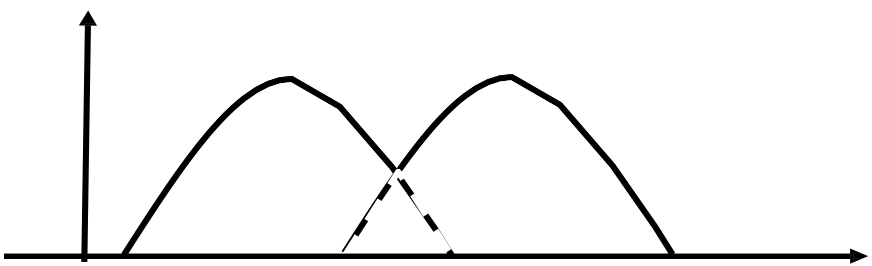
Operations of fuzzy sets:

Items	Set notation	Logic notation	Member. degree
Equivalence	$A = B$	$A(x) \Leftrightarrow B(x)$	$A(x) = B(x)$
Implication	$A \subseteq B$	$A(x) \Rightarrow B(x)$	$A(x) \leq B(x)$
Complement	$\overline{A}$	$\sim A(x)$	$\overline{A}(x) = 1 - A(x)$
Union, OR	$A \cup B$	$A V B$	$\max\{A(x), B(x)\}$
Intersection, AND	$A \cap B$	$A \wedge B$	$\min\{A(x), B(x)\}$
The complete law	$A \cap \overline{A} \neq \emptyset$ and $A \cup \overline{A} \neq X$		

Examples:







Upper-left: AND
Lower-left: OR
Upper-right: Negation

Soft Sets

Definition

Let X be an universe set and E be a set of parameters.

The soft set $F_E = (F, E)$ on X is a mapping $F: E \longrightarrow P(X)$ where, the value $F(e)$ is a subset of $P(X)$, that is a soft set on X can be represented by the form: $F_E = \{(e, F(e)) : e \in E\ ,\ F(e) \in P(X)\}$.

For example:

Let X is the set of six houses under consideration and,

E is the set of parameters, where,

$E = \{\text{expensive, beautiful, wooden, cheap}\}$.

- In this case, the soft set (F, E) describes the "attractiveness of the houses" which Mr. X (say) is going to buy.

Suppose that there are six houses in the universe X given by

- $X = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and $E = \{e_1, e_2, e_3, e_4\}$, where
- e_1: stands for the parameter 'expensive, '
- e_2: stands for the parameter 'beautiful, '
- e_3: stands for the parameter 'wooden, '
- e_4: stands for the parameter 'cheap, '
-

Now suppose that

- $F(e_1) = \{h_2, h_4\}$,
- $F(e_2) = \{h_1, h_3\}$,
- $F(e_3) = \{h_3, h_4, h_5\}$,
- $F(e_4) = \{h_1, h_3, h_5\}$. Then

The soft set (F, E) is a parameterized family $\{F(e_i): i = 1, .., 4\}$ of subsets of X and give us a collection of approximate descriptions of an object, and can be written as follows:

F_E ={expensive houses = $\{h_2, h_4\}$, beautiful houses = $\{h_1, h_3\}$, wooden houses = $\{h_3, h_4, h_5\}$, cheap houses = $\{h_1, h_3, h_5\}$ }.

Remark: For the purpose of storing a soft set in a computer, we could represent a soft set in tabular form and indicated that how it can be used in decision making.

U	Expensive	Beautiful	Wooden	Cheap	Sum	F-set
h_1	0	1	0	1	2	05
h_2	1	0	0	0	1	0.25
h_3	0	1	1	1	3	0.75
h_4	1	0	1	0	2	0.5
h_5	0	0	1	1	2	0.5
h_6	0	0	0	0	0	0

Tabular representing of a soft set

Fuzzy Soft Sets

Definition

A fuzzy soft set $f_E = (f, E)$ on X is defined by the set of ordered pairs $f_E = \{(e, f(e)) : e \in E, f(e) \in I^X\}$, where f is a mapping from E to I^X and I^X is the set of all fuzzy sets on X.

For example:

Consider the above example. The fuzzy soft set f_E can describe the "attractiveness of the Houses" under the fuzzy circumstances

$$f(e_1) = \{(h_1, 0.3), (h_2, 0.8), (h_3, 0.4), (h_4, 0.7), (h_5, 0.5), (h_6, 0.1)\},$$
$$f(e_2) = \{(h_1, 0.3), (h_3, 0.5), (h_4, 0.8)\},$$
$$f(e_3) = \{(h_4, 0.6), (h_5, 0.3)\}$$
$$f(e_4) = \{(h_2, 0.6), (h_4, 0.7), (h_6, 0.5)\}.$$

Then the fuzzy soft set (f, E) is a parameterized family $\{f(e_i): i = 1,2,3,4\}$ of fuzzy sets on X and give us a collection of approximate descriptions of an object in X.

Note: For the purpose interring data of a fuzzy soft set in a computer, we can represent it as in the below table

U	e_1=expensive	e_2=beautiful	e_3=woody	e_4=cheep	value
h_1	0.3	0.3	0	0	0.6
h_2	0.8	0	0	0.6	1.4
h_3	0.4	0.5	0	0.7	1.6
h_4	0.7	0.8	0.6	0	2.1
h_5	0.5	0	0.3	0	0.8
h_6	0.1	0	0	0	0.1

Tabular representing of a fuzzy soft set

Some tools and concepts in fuzzy system

- ## Fuzzy Rule

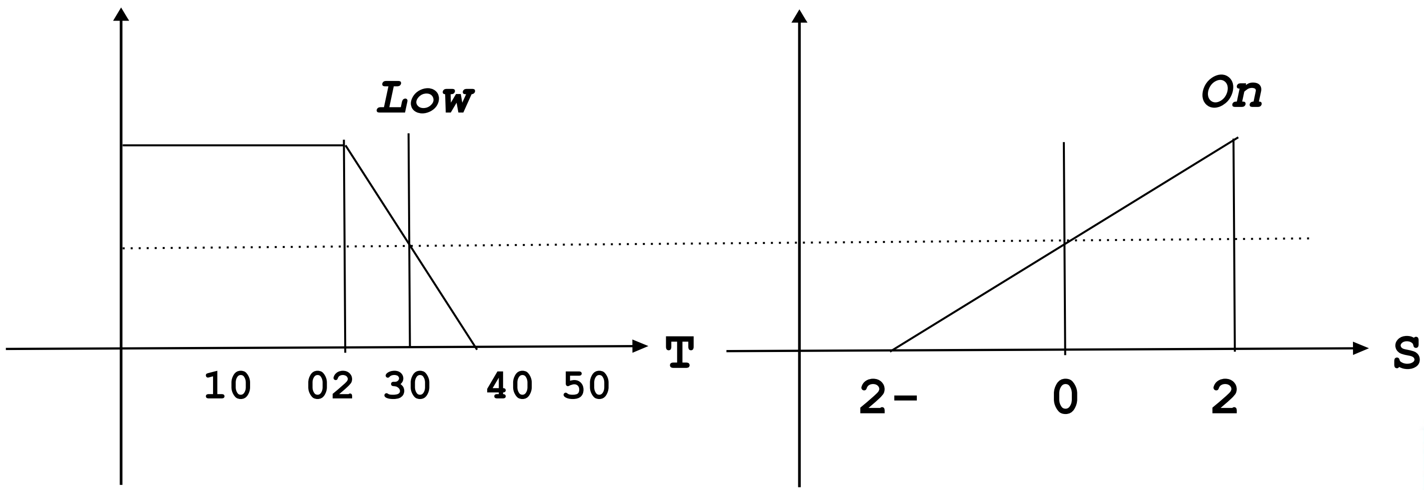
Let N: the number of attributes and, K: number of rules.

Then the $K-th$ fuzzy rule R_K is given as follows:

$$\text{If } (x_1 = F_{K_1}, x_2 = F_{K_2}, \dots, x_n = F_{K_n}), \text{ then } y = B_K$$

Where F_{K_n} , B_K are linguistic values.

Example of fuzzy rule



If (Water temperature = Low)
Then (Heating = On (

Pattern matching for fuzzy rules

- For a given input source $X = \{x_1, x_2, \ldots, x_n\}$ the similarity between x and the condition of the $K - th$ rule R_K is defined by:

$$\boldsymbol{S}(\boldsymbol{x}, R_K) = F_{K_1}(x_1) \wedge F_{K_1}(x_2\ ) \ldots (x\_1) \wedge F_{K_n}(x_n)$$

where $\wedge$ is defined as the

The similarity can also be considered the degree of matching

Fuzzy inference

For a given input source $x = \{x_1, x_2, \dots, x_n\}$ we can make a fuzzy decision as follows:

- Step 1: Find the membership function of the result B^*, which is also a fuzzy number, using the following equation:

$$B^*(y) = R_1(x) \vee R_2(x) \vee \cdots R_K(x)$$

where $\vee$ is max, and $R_K(x) = S(R_K) \wedge B_K(y)$

Fuzzy inference

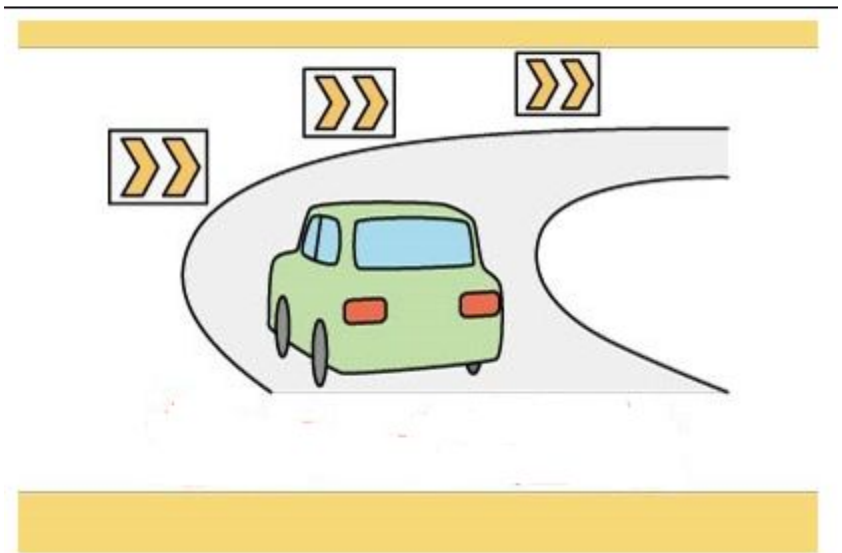
- Step 2: The final output is calculated as follows:

$$b^* = \frac{\int_Y y \times \mu_{B^*}(y)dy}{\int_Y \mu_{B^*}(y)dy}$$

  - This is the gravity center of the fuzzy number B^*.
  - We may also use median or the maximum value.
  - The process for finding a concrete number from a fuzzy number is called **de-fuzzification**.

Example

- According to Road laws, road around the corner must be constructed based on the relation between the curvature and the speed limitation.
- For example, if the speed limit is 50 km/h, the radius of the curve must be100 m or more



Example of Fuzzy rules for speed control

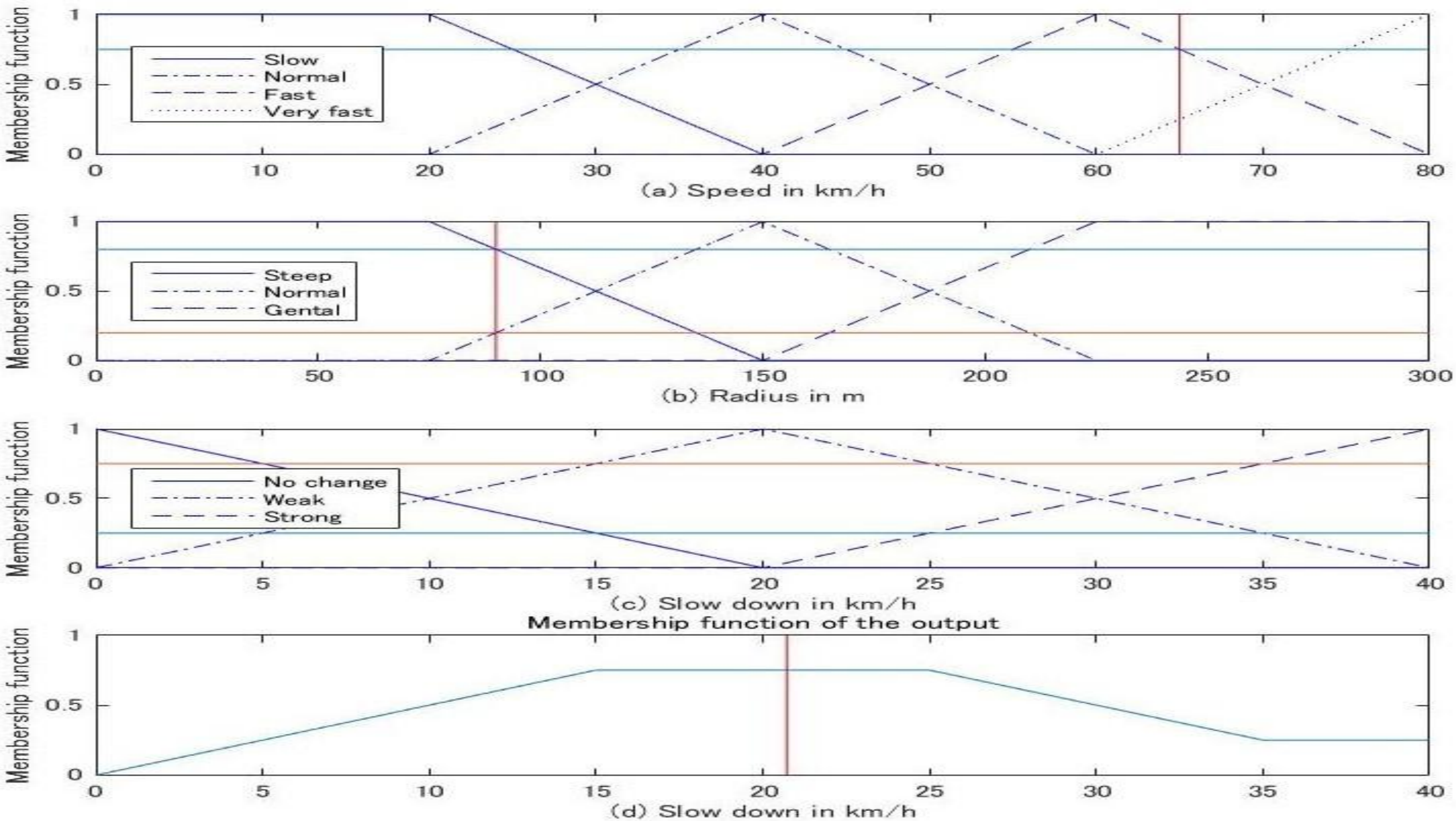
- R1: If(v=Normal ∧ r=Sharp curve(
Then (B=Weak(

- R2: If(v=A little fast ∧r=Sharp curve)
Then (B=Weak (

- R3: If(v=Fast ∧ r=Normal(
Then (B=Weak(

- R4: If(v=Fast ∧ r=Sharp curve)
Then (B=Strong(

Speed (v(	Curvature radius (r(	Break (b(
Normal	Slow curve	As is
Normal	Normal	As is
Normal	Sharp curve	Weak
A little fast	Slow curve	As is
A little fast	Normal	As is
A little fast	Sharp curve	Weak
Fast	Slow curve	As is
Fast	Normal	Weak
Fast	Sharp curve	Strong


Slow
Normal
Fast
Very fast
Membership function
(a) Speed in km/h
Steep
Normal
Gental
(b) Radius in m
No change
Weak
Strong
(c) Slow down in km/h
Membership function of the output
(d) Slow down in km/h

Problems in using fuzzy control

- Fuzzy control is a kind of soft control that uses human knowledge.
- The performance depends on the membership functions of the linguistic values.
- To find the optimal membership functions, we must use some other tools.
- For example, we may use genetic algorithm to fine-tune the parameters of the membership functions (e.g. to determine the shape or type(.
- We may also use deep neural networks to capture the best membership functions through learning.

Programming assignment

- Based on the skeleton program, write a program to implement a fuzzy system for speed control.
- Plot the results using “gnuplot”, and save the 3-D figure.
- The name file should be “plot.png”, and the coordinates are as follows:

) X, Y, Z) = (v, r, b(

- From the figure, try to describe the relation between b and (v, r), and add your description in “summary_09.txt”.

Thank you for

- IIE-SRF
- Cihan Univ-Erbil
- My colleagues at CSD